OPTICS
PAINTING &
PHOTOGRAPHY

M. H. Pirenne
Lecturer in Physiology, University of Oxford

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8 The perception of ordinary pictures

When viewed in the usual way with both eyes, ordinary pictures drawn in perspective—paintings in a gallery, posters, photographs, postcards, illustrations in books or newspapers—do not, whatever the sense of depth they may seem to give, appear in three dimensions like stereoscopic pictures.

Pictures seen with one eye only

Yet when they are viewed with one eye only, without moving the head, they may acquire this 'stereoscopic' appearance, even though the effect is unlikely to be as striking as in the case of Pozzo's ceiling. A painting may then appear, through its frame, as a scene in three dimensions seen through a window. Even photographs in a newspaper may appear in the same way, as if seen through a hole cut in the paper. This effect takes a little while to establish itself, rather like that given by the stereoscope. Schlosberg (1941) discusses the main types of experimental arrangements which can produce it. He points out its 'full or none' character: either it 'works', or it does not. Thus there is a sharp distinction between the usual mode of appearance of pictures and this three-dimensional appearance given by a single picture viewed with one eye: the picture becomes a kind of trompe-l'œil (see footnote 2, p. 93).

Assuming the eye to be at the centre of projection, the explanation is the same as for Pozzo's ceiling. As vision is uniconial, the flatness of the picture is no longer evident and the picture surface, not surface, is no longer seen. So perspective, both linear and aerial, colour, light and shade, the factors specific to pictorial representation, come into full play. The complex flux of light received from the picture by the one eye used is similar to

1 A painting by A. Wiertz (1866–69), 'The premature burial' (L'inhumation précipitée), showing a man trying to break out of his coffin inside a vault, used to be arranged in Brussels so that it only could be seen through a peep-hole in the wall of a room of the Museum. This manoeuvre picture then made a much more startling impression than when simply hanging on the wall, partly because it acquired a strong three-dimensional appearance. Similar arrangements have been used to create trompe-l'œil in experiments on visual perception (Pitton, 1949).
that which it would receive from the scene represented. Accordingly the picture appears as a scene in three dimensions.

Thus there is a sharp difference between the ordinary appearance of a picture and the appearance of the same picture under the special conditions just described. There is an even more striking difference between the usual appearance of ordinary pictures and that of Pozzo’s painted ceiling, as this painting looks strongly deformed, and yet in 3D, when seen from a wrong position.

**Ordinary pictures usually do not appear deformed**

When, as is most often the case, ordinary pictures are seen binocularly from a position different from the centre of projection, they do not as a rule give a noticeably deformed view of the scene represented. This fact, usually taken for granted, is a most important one in practical life. If it were not so, the usefulness of pictures as representations might almost vanish.

The kind of deformation which might be expected to appear when the picture is seen from the wrong position is illustrated by Fig. 8.1. The portrait of 'Big Brother', which appears in a poster in the background of this photograph, looks deformed because the photograph has been taken with the plate at an angle to the plane of the poster containing the portrait. Now the point is that portraits which are seen, not photographed, from a position well away from the centre of perspective of the portrait, do not look deformed as that of Big Brother in Fig. 8.1. If they did, painted portraits and photographs in passports, for instance, would be of very limited use, as they could give a good likeness only to one person at a time.

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1 The same effect may be produced by colour transparency photographs seen with one eye through a postcard lens, with the kind of viewer now in common use. As small transparencies require a lens in any case to be seen clearly, however, there is but little opportunity for comparison with the way they appear in binocular vision.

2aval, who was one of the translators of Helmholtz’s *Optics*, invented an instrument he called an *iconoscope*, which gives an effect similar to that which occurs in the binocular vision of ordinary pictures, even though binocular vision is used (Helmholtz (1867) *Optique physiologique*, pp. 822–3).aval’s iconoscope consists of a system of four mirrors, so arranged that the spectator sees with both eyes almost the same view as he would see with one eye placed midway between his eyes. This eliminates the stereoscopic element in vision by suppressing the binocular disparities of the visual angles.

3Leonardo da Vinci, on the basis of the fact that our two eyes give us two different views of the same scene, stated in his *Treatise on Painting* that it was impossible even for the most perfectly executed painting ‘to seem in the same relief as the natural model, unless that natural model is looked at from a great distance with one eye’ (McMahon (1966), Vol. 1, p. 177). Yet he seems to have noticed the peculiar three-dimensional appearance which ordinary pictures can acquire when viewed with one eye only (Richer & Richer (1930), No. 59, see Italian text).

Leonardo’s observations on binocular vision might conceivably have led him to invent the iconoscope. But in fact it was only invented in 1833, by Wheatstone; see Helmholtz (1860), p. 690; Asher (1961).

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8.1 Deformation in a photograph of a photograph

The head of ‘Big Brother’ in the background looks deformed because it is a photograph, taken at a wrong angle, of another photograph. The head of the young man in front, on the other hand, does not look deformed when seen from a position away from the centre of projection of the photograph. See text. (From the *Radio Times*, 25 November 1965, ‘The World of George Orwell: Nineteen Eighty-four’.)
In Fig. 8.1, itself, the picture of the young man in front of the poster hardly becomes deformed when viewed from a wrong position. The same would no doubt be true of the actual poster portrait of Big Brother, if the poster itself were seen in its own frame, whereas in Fig. 8.1 we only see a photograph of the poster, without its frame.

Similarly, in Fig. 8.2, the portrait in the background is deformed into something of a caricature, because it has been photographed at an angle. Yet, while most spectators attending the electoral meeting at which this large portrait was displayed were viewing it from a wrong position, they must hardly have noticed deformations of this kind—which would make the use of such portraits defeat its purpose.

When the shape and position of the picture surface can be seen, an unconscious intuitive process of psychological compensation takes place, which restores the correct view when the picture is looked at from the wrong position. In the case of Pozzo's ceilings, on the other hand, the painted surface is 'invisible' and striking deformations are seen. Again, striking deformations are seen in the case of anaglyphs, a stereoscopic device in which the scene represented is seen in three dimensions; and the picture surface again is 'invisible' (see Chapter 11).

The existence and importance of this concept of psychological compensation were conveyed to the present writer by a letter from Albert Einstein, dated 24 February 1955, in which he wrote:

"The perspective representation corresponds exactly to the optical impression given by the object for a certain position of the eye in front of the plane of projection (picture).

When one views the picture on its own, but from a different centre, then one receives visual impressions which the object itself would not give. This will almost always be so, when one looks at a painted picture.

'Now it seems to me to be the case that the spectator will easily compensate intuitively for this deformation, when the angle $\alpha$ which defines the boundary of the object is small, so that all the visual rays strike the plane almost perpendicularly [Fig. 8.3]. When, however $\alpha/2$ is for instance equal to $45^\circ$, then the picture will appear deformed, if it is viewed from a distance which is greater than the distance of the projection centre from the picture. Then the intuitive compensation is likely to fail, so that the picture appears deformed. It is not easy to say which compromise solution the painter must choose in such a case.'

'It is probable that the use of a sufficiently restricted angle $\alpha$ in combination with central projection, is the only reasonable solution."

The whole passage of Einstein's letter which relates to perspective reads as follows in the original:

Die Frage betreffend der Perspektive im Zusammenhang mit der künstlerischen Darstellung erscheint mir doch noch problematisch. Die perspektivische Darstellung ist dem optischen Eindruck des Objektes genau entsprechend für eine bestimmte Lage des Auges gegenüber der Projektions-Ebene (Bild).

Wenn man das Bild allein betrachtet, aber von einem andern Zentrum aus, so erhält man
Einstein does not state explicitly that the basis for this compensation must be the spectators' awareness of the characteristics of the surface of the picture. But it can hardly be anything else. Paradoxically this awareness must be helped by the use of binocular vision, even though the picture in perspective is a projection made for one eye only.

Gestalt-Wahrnehmungen wie die des Objekts nicht vermittle. So wird es nahezu immer sein, wenn man ein gemaltes Bild betrachtet.

Es scheint nun so zu sein, dass diese Abweichung von dem Beschauer des Bildes leicht intuitiv kompensiert wird, wenn der die Objekt-Begrenzung charakterisierende Winkel klein ist sodass alle Schatten die matte nebene senkreche treffen. Wenn aber 2/2 z. B. 45° ist, dann wird das Bild verzerrt aussehen, wenn man es von einer Distanz betrachtet, die z. B. wesentlich größer ist als die Distanz des Projektioms-Zentrums von Bild. Dann wird wohl die intuitive Kompensation versagen, sodass das Bild verzerrt erscheint. Es ist nicht leicht zu sagen, was für ein Komprromiss der Maler in solchem Falle am besten wohltun soll.

Wahrscheinlich ist die Beschränkung auf hinreichend kleine in Kombination mit der Zentralprojektion die einzige vernünftige Lösung. Dann ist Ihre Behauptung im Wesentlichen richtig.

The first sentence ("The question concerning perspective with regard to representation in art does seem to me still unsolved") and the last sentence ("Then your thesis will essentially be correct") refer to a paper by the present writer published in 1962, now partly superseded, on which he failed to take into account the process of compensation which has just been discussed. A great deal of the present book consists of an examination of problems suggested by Einstein's letter. (The main thesis of the book was outlined by the author in 1963.)

8.4a, b Two pinholed photographs of the same statuette taken from different distances. Photograph 8.4a was taken from a distance equal to half the height, photograph 8.4b from a distance equal to three times the height of the statuette. See text.

Bronze statuette stamped De Luca, Napoli. Reduced and restored copy of Aphrodite, No. 2021, British Museum Catalogue of Bronzes, published in H. B. Walters (1915), British Museum, Greek, Roman and Etruscan, in the Department of Antiquities. London: British Museum, Plate XXVII. Said to have been found near Patras (perhaps at Olympia). Probably second century B.C. Information kindly given by Mr. M. S. Higgins.

8.3
The influence of the choice of the angle $\alpha$ is illustrated by the two photographs of the same statuette reproduced in Figs. 8.4a and b. In Fig. 8.4a the distance of the projection centre to the statuette is equal to about one-half of the height of the statuette, so that $\alpha$ is about 90° (the distance of the projection centre to the photograph is too short to make it possible to see it clearly from this position). Artists and photographers have usually avoided the use of such a short distance for figures and portraits.

8.5a View of Venice by Canaletto 1697-1768
London: National Gallery.

In Fig. 8.4b the distance of the centre of perspective is three times the height of the statuette, the angle $\alpha$ being equal to about 20°. The classical proportions of the statuette here are retained, whereas they appear distorted in Fig. 8.4a. Moreover the picture gives a good representation of the statuette for any position in which the spectator is likely to place himself.

When on the other hand the angle $\alpha$ is very small, the picture approximates to an orthogonal projection—which may be regarded as a central projection from a centre so distant that the rays of the visual pyramid can be taken as being practically parallel. If

8.5b Enlargement of part of Canaletto's painting Fig. 8.4a
This is a distant view covering a small visual angle. It corresponds to what would be seen with a telescope. See text.

the view is a distant scene, the objects depicted then appear ‘squeezed’ together, like the distant squares in the retinal image shown in Fig. 6.1. The picture then resembles what is seen in a telescope, or photographed by a telephoto lens (Figs. 8.5a and b).

Different projections of the same view

The two photographs of the Arch of Janus in Rome, Figs. 8.6a and b, were taken with a pinhole camera, both in the same manner, except that the plate of the camera was parallel to the plane of the façade for Fig. 8.6a, whereas for Fig. 8.6b it was placed at an angle of
25° to the façade. The camera was not displaced bodily from one position to another. Using a special arrangement, it was merely rotated by 25° around a vertical axis passing through its pinhole, the position of the centre of the pinhole thus remaining quite unaltered. Two accurate central projections of the view containing the Arch were therefore obtained from the same centre of projection \( O \) on two vertical planes, \( AB \) and \( A'B' \) (Fig. 8.6c), the only difference between the two being that the second vertical plane \( A'B' \) of projection was at an angle of 25° to the first plane \( AB \), as shown in the diagram.

To make this quite clear imagine that a spectator places his eye so that its centre of rotation is at \( O \), at the centre of projection, while keeping his head motionless (Fig. 8.6c). He then sees a certain view of the scene in front of him. Since he does not move about, his standpoint defines this view, which corresponds to the whole pyramid of sight relating to his eye’s position. He can turn his eye and look in various directions, but what he sees is always part of the same view. Let the spectator now imagine in front of him a vertical sheet of glass, that is, a Leonardo window, placed in the position \( AB \), on which glass pane he draws the Arch in such a manner that the drawing covers point by point the view he sees through the pane. This will give one section of part of his pyramid of sight, which will be like the photograph of Fig. 8.6a. Another drawing of the same view made in exactly the same way, again from \( O \) as a centre, but on a sheet of glass in the position \( AB' \), will give a different section of the same pyramid of sight, corresponding to the second photograph, Fig. 8.6b.

By changing only the orientation of the surface of projection, any number of different photographs, or perspective drawings, can therefore be made of the same identical scene from exactly the same centre of projection. Yet when the centre of rotation of the spectator’s eye is kept fixed in this projection centre, what he sees is always the same view. As a rule, only trained artists and photographers clearly realize the different results which will be obtained if this same view is projected onto planes differing in their orientation.¹

¹ This discussion strictly refers to vision with only one eye, the centre of rotation of which is in a fixed position. In effect, except when objects very close to the eyes are involved, it is also approximately valid when the spectator, standing in one position, looks around with both eyes.
On account of the limited size of the photographic plate, Fig. 8.6a does of course show more of the view on the right of the Arch than Fig. 8.6b, and vice versa. The two photographs of the Arch itself, which is seen as a whole in both Figures, do, however, represent exactly the same view of it. What is seen in one photograph is seen in the other. What is hidden in one photograph is hidden in the other. This is so for the architectural details just visible inside the right side of the passage through the Arch. It is only by bodily displacing the camera, or the eye, onto a new position towards the left, that more details

8.7a, b Two pinhole photographs of the so-called Temple of Neptune at Paestum
The photographs were taken in the same way as Figs. 8.6a and b, but the angle of the planes of projection was 30° instead of 25°. This Greek Doric temple at Paestum, south of Naples, was probably built c. 460 B.C., in the same century as the Parthenon (447–438 B.C.). It is now believed to have been dedicated to Hera, not to Poseidon.
8.8a, 8.8b  Two pinhole photographs of the Temple of Neptune at Paestum, taken on a vertical and an inclined plate.

The plate for 8.8b was inclined at 20° from the vertical, the pinhole being in the same position for 8.8a and b.
could be seen on this side of the interior of the Arch; this, of course, is simply due to the fact that we cannot see round corners.

Yet the two photographs look very different. One is a frontal perspective. The other is a perspective taken at an angle, showing considerable foreshortening of the façade. Figure 8.6a, the façade appears like an architect’s elevation; horizontal lines in it are horizontal, arcs of circle belonging to it are circular. In the other, Fig. 8.6b, the façade of the Arch is foreshortened; horizontal lines in the architecture all converge towards a point on the left; arcs of circles in the plane of the façade have become elliptical in shape. Indeed it may be said that Fig. 8.6a is composed like an Italian painting of the Quattrocento, while Fig. 8.6b is more Baroque in style.

The two photographs, Figs. 8.7a and b, of the so-called Temple of Neptune at Paestum, were obtained in a similar way. The centre of projection is exactly the same for both. Only the orientation of the two vertical planes of projection differs, by an angle of 30°.

In the two photographs, Figs. 8.8a and b, of the same Greek temple, the centre of projection is again the same. The plane of projection is vertical in Fig. 8.8a. But in Fig. 8.8b it has been rotated by an angle of 20° around a horizontal axis. The result is that vertical lines now appear to converge upwards, whereas they are vertical in Fig. 8.8a. (In fact these two photographs were again taken from the same centre of projection as the preceding pair, so that Figs. 8.7a–8.8b give three different projections all of the same view—Figs. 8.7b and 8.8a give essentially the same projection.)

**Experiment**

A similar pair of photographs (Figs. 8.9a and b) was made for a set of metal cubes and cylinders arranged as shown in the diagram of Fig. 8.9c. The set of metal objects is shown on the right of the diagram. The camera is shown on the left, its pinhole being at O. The photograph of Fig. 8.9a was taken with the plate AB parallel to the front of the set of objects; that of Fig. 8.9b was taken with the camera rotated by 20° around a vertical axis passing through O, the plate being moved to the position A'B'. The point O remained the centre of the pinhole of the camera for both pictures. Thus the two central projections of the same set of objects, Figures 8.9a and b, have the same centre of projection.

Enlargements of the two photographs of Fig. 8.9a and b were viewed, one at a time, through a hole 1 mm in diameter (Fig. 8.9d). Each photograph could be exactly positioned so that this small hole, or artificial pupil, was at the centre of projection O of the photograph. Furthermore, as shown in the diagram of Fig. 8.9d, the same vertical screen LM, pierced with a rectangular aperture, was interposed between the artificial pupil at O and the photograph, each photograph in turn being placed at the correct position. Under these conditions the observer viewing either photograph AB or photograph A'B' through the artificial pupil and through the aperture in the screen LM was unable to detect any significant difference between the appearance of one and the other. Both acquired a depth and relief similar to that given by stereoscopic pictures. The spectator no longer saw the surface of the picture, qua surface, and had no inkling of its orientation relative to his eye. (The small artificial pupil was used in order to give to all observers a clear view of the photograph, as vision through such a small hole gives a very great depth of field to all eyes (see Scheiner’s experiment in Chapter 1).)
In this experiment, therefore, the two photographs appeared in three dimensions (like Pozzo's ceiling) and they could hardly be distinguished from each other. The photographs had been positioned so that all the visual angles subtended by their various parts were the same as the angles which would be subtended by the corresponding parts of the objects themselves. This result again shows that pictures in perspective may, when viewed under special conditions, be seen as predicted by the simple theory of linear perspective. In agreement with this theory, different projections of the same objects, that is, different sections of the same visual pyramid, made and viewed from the same centre of projection, may appear the same to the eye when special arrangements are made. (The same result was obtained using two photographs of a set of objects consisting of 12 square pillars; Pirenne, 1967b.)

Essentially the same experiment was made with Figs. 8.8a and b, the plane of Fig. 8.8b being inclined at 20° from the vertical. Both photographs gave the same view, in 3D, of the temple. Held in the hand, on the other hand, Fig. 8.8b rather looks like one of those crazy pictures of skyscrapers, and most observers agree that it cannot really be made to look like Fig. 8.8a. In such a case, it will be noted that no mask is used and that the boundary of the photographs is seen, which gives clues as to the position of their surface.

The spectator's awareness of the surface pattern of ordinary pictures

It has often been taken for granted that using one eye only, and holding by hand in the right position pictures like the pairs of photographs just discussed, they can all be made to look the same without any special difficulty (the present writer used to think so, but had to change his mind once he had made the above-described experiments). In fact this is not so. The pairs of photographs, Figs. 8.6a and b, and 8.8a and b, cannot easily be made to look the same in this informal way. It is true that the architectural views are wide-angle photographs, the height of Fig. 8.8a and b for instance subtending about 96° at the centre of projection, so that the centre of projection is usually too near the picture for clear vision. But even in the case of enlargements which can be seen clearly from their centre of projection, the two photographs of each pair do not look quite the same when held in the hand, even when they are viewed with one eye only.

Viewed with both eyes, in the usual manner, these photographs, as well as paintings in perspective, give a certain sensation of solidity and depth, even though not a 'stereoscopic' one. But, on the other hand, in Figs. 8.6a and b and in Figs. 8.9a and b, for instance, the spectator also recognizes either a frontal perspective or a perspective taken at an angle.

It may be concluded that under ordinary conditions the actual pattern on the surface of a representational picture must be perceived, as a surface pattern, even though the
spectator may only be dimly aware of this, at the same time as the objects represented are seen as a scene in depth.

The main point of the above experiments is that they made the surface of the picture 'invisible', for surface. The frame of the picture itself was no longer visible, being hidden by the new 'frame' given by the aperture in the screen placed between the artificial pupil and the picture. In ordinary vision the spectator's awareness of the surface pattern is obviously helped to a great extent by the frame or by the regular shape of the boundary of the picture.

The awareness of the surface pattern of a painting is clearly of great importance from an aesthetic point of view. The artist or the photographer must compose the scene he wishes to represent, by deciding from which standpoint he is going to depict it. But the choice of the surface of projection for this perspective of the scene and of the boundary or frame of this surface also are of decisive importance for the composition. For instance, in a frontal perspective of a façade the horizontal lines are parallel to the top and bottom of a rectangular frame. In a perspective taken at an angle, they are no longer so. This, from an artistic standpoint, is of major importance.

'There is a kind of emotion which is quite particular to painting,' wrote the painter Delacroix, 'and which results from such and such arrangement of colours, lights, etc. That is what one may call "the music of the painting".' Even before knowing what the painting represents, being at too great distance to see what it represents, often you are taken in by this magical accord.' (Roger Marx, 1962) Such an aesthetic effect can only be produced by the harmonious characteristics of the flat pattern of the surface of the painting, since it occurs before the scene represented can even be identified. The fact that this pattern is in fact also perceived to some extent at close range, when the scene represented is clearly seen, shows that representational painting does not differ radically as is sometimes believed from non-representational, 'abstract', painting, in which only that flat painted pattern is often considered as being of importance.

An unsophisticated spectator may entirely fail to appreciate the artistic worth of a purely non-representational design. But in front of a representational painting by a great artist, he may be influenced, quite unwittingly, by the purely aesthetic character of the surface pattern of the picture, even while he may think he is only interested in the scene represented.

Our awareness of the characteristics of the surface of a picture may be called a 'subsidiary' awareness, in the sense in which Polanyi (1958, 1962) uses this term, in contradistinction with our 'focal' awareness of the subject represented. At the same time as we are attending to the scene in depth, we are also aware, in a much less positive way, of many clues relating to the surface and the frame of the picture.\(^1\)

\(^1\) This applies to the case of a spectator whose major interest lies in the scene represented. The connoisseur and the expert can transfer his focal awareness from the scene to the painted surface,
9 Objects with curved surfaces

Spheres and human figures
The outline of a sphere always appears circular to the eye, for the cone tangent to the sphere and having its apex at the point in the eye always is a right circular cone. Now the section of such a cone by a plane is a circle only when the axis of the cone, which passes through the eye $O$ and through the centre of the sphere, is perpendicular to the intersecting plane $AB$ (Fig. 9.1). When the cone is cut by a plane $AB'$ oblique to its axis, the angle of the axis to the plane being smaller than the angle of the cone, the resulting section is an ellipse.¹ Thus in a picture in exact central perspective, the projection of a sphere should in most cases be an ellipse. An instance of this is given by the pinhole photograph of a stone sphere on the right of Fig. 9.2. It may be emphasized that when the centre of rotation of the eye is at the centre of projection, then for each and every position of the eye the retinal image of the ellipse, assumed to be drawn on a Leonardo window, will coincide exactly with the retinal image formed by the actual sphere—whereas a circular image on the window would not do so. The spectator, without moving the head, is free to look at each part of the sphere or of its elliptical image in turn. When his eye thus rotates in its orbit, both retinal images may become altered; yet they always remain in coincidence.

The axes of all ellipses such as those in Fig. 9.2, when produced, pass through the point (the "principal point") where the perpendicular from the centre of projection intersects the plane of projection. The eccentricity and the orientation of the elliptical projection of a sphere vary with the position of the sphere in the picture. This is shown in Figs. 9.3 and 9.4 which are pinhole photographs, and therefore accurate central projections, of a set of five and a set of fifteen spherical objects (see Fig. 9.5). In each figure, the central sphere, which has its centre on the perpendicular from the eye to the picture plane, is the only sphere which is projected as a circle.

When an enlargement of the photograph of Fig. 9.3 is viewed through a small artificial pupil placed at the centre of projection of the picture, and through an oval aperture in a screen which hides the edges of the picture, most observers report that it looks like a set of equal spheres, and cylinders, seen in three dimensions. Such an arrangement is similar to that of the experiment described above for the set of cubes and cylinders in Fig. 8.9d. Here again the simple theory of perspective works as predicted; for under

¹ This is the usual case in the practice of perspective. For angles of intersection equal to or greater than the angle of the cone, the section is a parabola or a hyperbola.
9.2 Pinhole photograph of spherical architectural ornaments

The projection of the large sphere is elliptical in shape and the great axis of the ellipse is directed towards the principal point of the perspective. Picture taken on the roof of the Church of St. Ignazio in Rome.

9.3 Pinhole photograph of a set of five spheres on top of five cylinders

The objects photographed were aluminium cylinders and steel balls, all one inch in diameter. The picture was taken as explained in Fig. 9.5. (From Pirenne (1967 a), Vision and the Eye.)

9.4 Composite pinhole photograph of fifteen spheres on top of cylinders

The objects were the same as for Fig. 9.3. The arrangement is explained in Fig. 9.5. (From Pirenne (1967 a) Vision and the Eye.)
from which it is seen as a set of true spheres. Using binocular vision, this is impossible. The so-called marginal distortions in the photograph then always remain very conspicuous—these 'distortions' of course are in fact exact central projections.

Now, most artists have always depicted spheres as circles, not as ellipses. Italian artists of the Renaissance thus 'corrected' the 'distortions' shown in Figs. 9.3 and 9.4. It is true that Leonardo da Vinci (McMahon, 1956, Vol. 1, p. 92; Vol. 11, p. 49v) knew that the projection of a sphere in most cases is an ellipse; but, according to La Gurrerici (1859), he never depicted a sphere in any of his paintings. In his School of Athens, Raphael (1483-1520) depicted two spheres at the right side of his painting (Figs. 9.6 and 9.7). But he drew their outlines as circles. The architecture which extends over most of the painting is drawn in perspective as one whole, from one main centre of projection. But the spheres (and the numerous human figures) are not drawn as projections from this centre. They are drawn from a number of subsidiary centres of projection, each in front of the position which the respective sphere or figure would occupy in the painting.
This is an instance of the modifications made to the strict theory of central projection, even by artists who paid great attention to linear perspective. In the case of Raphael's spheres, La Gournerie relates in his treatise (1839, p. 170) that he made the following experiment. In an engraved reproduction of the School of Athens, he replaced the circular

pictures of the two spheres by ellipses having the correct, rather elongated, shapes which correspond to the position of the spheres relative to the main centre of projection. (The distance of this projection centre to the picture plane, in this painting, is about equal to the width of the painting.) These correct central projections, producing the same retinal images as would be given by actual spheres, were found to be quite unacceptable. They did not at all look like spherical objects—whereas Raphael's circles do so.

Thus it appears that the spectator looking at Raphael's picture of the spheres must make a complicated intuitive compensation. On account of natural perspective, the circles appear foreshortened to him. They do not form in his eyes the retinal images which would be formed by actual spheres. But, on the basis of his knowledge of the shape and position of the surface of the painting, he recognizes them as circles drawn on a flat surface. Since real spheres always look circular, he concludes that these circles represent spheres. It will be noted that all this, which must somehow occur unconsciously, can be done as well when the spectator uses both eyes, and is in the wrong position. To most spectators, the School of Athens, in which the perspective is in parts inaccurate, appears as an outstanding example of the use of perspective. Indeed it is wide-angle photographs, in which the perspective is accurate in all particulars, which on the contrary strike many as containing 'distortions'.

Human figures are usually depicted in the same manner as Raphael's spheres, from different subsidiary centres of projection placed each in front of the position in the painting of the figure concerned. This is so in the School of Athens. The artist has painted the various figures rather as if he had done their portraits, each portrait being placed in a separate part of the painting. It will be noted that for the artist this is the most natural and easy way of painting such figures. It is again in the case of wide-angle photographs that human figures, near the edge of the picture, often appear 'distorted'. The heads of the figures appear elongated along various directions which depend on their position relative to the point where the perpendicular from the centre of projection strikes the picture plane, as in the case of the spheres of Fig. 9.4. Artists whose aim is a good likeness do not draw figures as shown in the photograph of the Greek statuette, Fig. 8.4 a, which gives an exact projection. (Pozzo drew accurate central projections, but his paintings were of a very special kind.)

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1 The camera obscura, fitted with a lens, and with a plane mirror, which gives an upright image on a horizontal surface, was used by artists in the eighteenth century. Several types of these 'optical machines for forming the pictures of objects, with their use in drawing' are described by R. Smith in his Complete System of Opticks (1738), Bk iii, Ch. 15. Giossef (1749) has built and experimented with yet another type of camera obscura, known to have been used in the eighteenth century, and which was large enough for the artist to work with the upper part of his body inside the camera, stray light from outside being excluded by the use of a curtain. Such cameras produce a sheet of paper a luminous image of external objects, similar to the image cast by a photographic camera on the ground glass screen. The artist is thus able to trace on the paper the outlines of objects in the image produced by the camera, which is a central projection of the external scene. There are very strong reasons to believe that Canaletto (1697–1768) made at least some of his townscape drawings with the help of such a camera. According to Giossef, the dome of a church in one of Canaletto's drawings is asymmetrical with regard to the vertical, a kind of 'distortion' which is typical of photography and of perspective drawn with rigorous accuracy, but which, he says, would be 'inconceivable' in the case of a free-hand drawing executed by eye direct from the real object. (Yet in the background of Mantega's (1431–1506) painting of 'Christ in the Garden of Olives', which is in the Musée des Beaux Arts of Tours, there is a round building the dome of which is drawn in such an asymmetrical manner.)
Rows of columns

Figure 9.3 shows that in a row of cylindrical objects parallel to the plane of projection, the cylinders further away from the centre of projection give projections which become not thinner, but wider. The diagram of Fig. 9.5 explains this apparent paradox. The angle subtended at the centre of projection by the width of the cylinders does become smaller for more peripherally placed cylinders, but the section by the plane SS becomes wider because of the increasing obliquity of this plane. In the experiment with the artificial pupil described above with regard to the spheres on top of the cylinders, the cylinders also look like true cylinders all equal in size.

In Fig. 9.8 it will be noticed that the projections of the Doric columns become thicker towards the right and the left, for the same reason. When representing such a row of columns, artists have usually avoided this effect and

9.8  Pinhole photograph of columns of the so-called Basilica at Paestum
     Frontal perspective. The 'Basilica' is a Greek Doric temple of the sixth century B.C.

9.9  Another pinhole photograph of the Basilica at Paestum
     Perspective at an angle. The centre of projection was the same as for Fig. 9.8.

9.10 Engraving of the cloister of the Charterhouse at Rome
     From Letarouilly (1840-57), Edificios de Roma Moderna.
9.11a  Elevation and plan of the set of eighteen cubes, two perspectives of which are given in Figs. 9.11b and c.

Both perspectives are made from the centre of projection O, the first on a vertical plane AB parallel to the faces of the cubes, the second on a vertical plane CD at an angle of 15° to AB.

The angle a is about 66°, so that if the set of cubes were extended to the left to cover the same angle as to the right, Einstein's angle a would be 133°. This leads to strong 'marginal distortions' in the two central projections of Figs. 9.11b and c.

given equal width to all the columns. Similarly they have deviated from exact central projection when depicting a row of columns at an angle to the picture plane, thus avoiding the rather puzzling effect shown in Fig. 9.9, where the more distant columns on the left seem to become thicker, relative to their height, than those in the middle of the photograph. Thus in the architectural drawing of Fig. 9.10 by Lefrarguillu, the last column on

9.11b  Central projection of the set of eighteen cubes

The projection was made from O onto the plane AB. The projected faces of the cubes are square, as in the elevation of Fig. 9.11a.

This figure, as well as Figs. 9.11c, 9.12b, c and 9.13b, were produced at the Computing Laboratory of Oxford University by Dr M.D. McIlroy and Mr J.E. Scaggs using an English Electric KDF 9 computer and a Calculm plotter. (Most of the work was programmed in FORTRAN; the basic graphic routines were adapted from a Culham Laboratory package. See F. M. Larkin, A User's Guide to the Culham Graphical Output System, H.M. Stationery Office, Code 913/3/21/75.)

9.11c  Another central projection of the set of eighteen cubes of Fig. 9.11b.

The projection here was made from the same centre of projection as for Fig. 9.11b, but on a vertical plane inclined at 15° to the plane containing the faces of the cubes. (Picture made by computer as Fig. 9.11b.)
the right is the thinnest, and the second from the right is thinner than the other two on the left.

A row of square pillars does not give rise to the same problem. When projected on a plane parallel to the plane in which they lie, the rectangular fronts of the pillars all give

projections of the same shape and size. This is because the projection of a plane figure onto a plane parallel to it gives a figure which is geometrically similar to the original. The problem arises for columns because their surfaces are curved.

Figures 9.12b-9.13b show the difference between the central projections of sets of cubes, which may be taken as representing square pillars, and sets of cylinders representing columns. These figures show the strange perspective effects given by columns placed at large angles from the principal point, that is the point of intersection of the perpendicular from the centre of projection to the picture plane. The effects shown here are extreme cases of those ‘distortions’ which many artists have either ‘corrected’, or avoided altogether.
9.13a Elevation and plan of another set of eighteen cylinders, the perspective of which is given in Fig. 9.13b.
The cylinders here are more widely spaced than in Fig. 9.12a, but the cylinders marked 1-4 are common to both sets. The total angle subtended by the projection from O is about 126°, which leads to very strong marginal 'distortions' in the central projection on the vertical plane CD.

Balusters and similar objects
The surface of the curved part of a baluster is a surface of revolution, having a vertical axis. Accordingly, it always appears to the eye as symmetrical relative to a vertical plane passing through the axis and through the eye. Apart from exceptional cases, there is no such symmetry in the central projection of a baluster on a vertical plane. Figure 9.14 shows this clearly for the balusters distant from the centre of the picture; the deviation from symmetry is small for the two central ones.

9.13b Central projection of the set of cylinders of Fig. 9.13a
The widths of the projected cylinders pass from a very large value on the left through a local minimum, then a local maximum, and would finally tend to zero towards the right if the number of 'columns' were extended indefinitely. Measured on an arbitrary scale, the 8 projected widths in the Figure are about 2.45; 1.00; 1.45; 1.50; 1.54; 1.35; 1.31; and 1.22. (Picture made by computer as Fig. 9.14b.)
Artists have again avoided this effect in their painting, departing from exact perspective in order to draw symmetrical outlines of the balusters. The same applies to vases, lamps, cups and other solids of revolution (Figs. 9.15 and 9.16).

9.14 Pinhole photograph of a balustrade on the roof of the Church of St Ignazio in Rome.
Note the ‘distortions’ of the shape of the balusters away from the centre of the picture. The perspective construction of the straight lines in this photograph is analysed in Fig. 10.2 of next chapter.

Conclusion
In the case of almost all objects with curved surfaces the shape of which is well-known to the spectator, artists have usually modified the exact rules of perspective which of course imply the use of a single projection centre. This is in contrast to the warning given by Pozzo with regard to the construction of his ceiling in perspective, namely that figures and architectural features there must all be drawn in accurate central projection from the centre of projection indicated by the marble disc in the floor of the church. While Pozzo aimed at and achieved a grandiose trompe-l’œil effect, however, artists making ordinary paintings did not aim at such an effect, and must have expected their pictures to be viewed from various positions—under which conditions the spectators are aware of the position, shape, and other characteristics of the surface of the picture.

9.15 Banqueting scene. Painting from Pompei.
The tops of the small table and of the cups placed upon it are drawn more or less as ellipses, but the great axes of these ellipses are nearly horizontal, not showing the inclinations which appear in Fig. 9.16. (Another painting at the Naples Museum, Inv. No. 9021, shows a similar depiction of a number of round objects, but is seriously damaged.) (Museo Nazionale, Naples. Inventory No. 9024.)

The above modifications to central projection are not always easy to harmonize with the perspective of the straight parts of the objects, drawn in accurate projection from the main centre of projection. Difficulties arise notably with regard to curved mouldings in the bases and capitals of columns. Figures 9.8 and 9.9 show the strange shapes which may be assumed by the circular tops of Doric columns in central projections.
It would appear, in fact, that many Italian painters of the Renaissance avoided such architectural features, or at any rate did not make them play a major role in their composition. The engravings in treatises on perspective also contain but few such instances, no doubt because exact perspective in such cases leads to results which experience taught artists to avoid as odd and displeasing. La Gournierie (1859) deals in detail with this problem. He points out that most treatises in perspective published before his own fail to discuss or even mention the modifications which experience led artists to make to the rules of exact perspective.

The Dutch painter Saenredam (1597–1665), however, did put to artistic use some of the perspective effects which have just been discussed. In Fig. 9.17, some of them appear in the base of the central column of the painting. The principal point is far to the left (as in Figs. 9.17b and 9.12b) and φ/2 is also a large angle in this picture. It appears that Saenredam increased the height of the near column, thus changing the architecture of the church, so as to avoid still stranger effects which otherwise would have occurred with regard to the tops of the columns. It will be noted that these effects only occur in the perspective, that is, in the section of the visual pyramid. They are not seen when one looks at the real architecture (see Carter, 1967).
Central projection of straight lines. Subjective curvatures

The central projection of a straight line is reduced to a single point if the line passes through the centre of projection, that is, through the centre of rotation of the eye. Then the straight line is a line of sight, all the points of which have projections in coincidence with one another—and have coinciding retinal images, as shown in the experiments on the rabbit's eye. This applies to surfaces of projection having any shape.\(^1\)

When the surface of projection is a plane, a straight line which does not pass through the centre of projection is projected as a straight line. For its projection is the intersection of the plane defined by the line itself and the centre of projection, with the projection plane; this intersection of two planes is, of course, a straight line.

Advantage of a plane surface of projection

Thus, in a plane picture, all lines which in the scene depicted are objectively straight are projected as straight lines, except when they are foreshortened into single points. Consequently, when the picture is seen from a position different from the centre of projection, the straight lines belonging to the objects represented will still be seen as straight lines, even if the shape of the objects then take up a deformed appearance.

This is one of the main advantages of pictures having a plane surface. The position is different in the case of central projection on a curved surface, when it becomes much more important to keep the eye at the right position.

Suppose an architectural view is projected on the inside of a spherical surface, the centre of the sphere being the centre of projection. Then all straight lines of the objects (except those passing through the centre, which are projected as single points) are projected as segments of great circles on the sphere. Seen with one eye placed at the centre of the sphere, these great circles will appear as straight lines, at any rate in the sense that it will be possible to cover them exactly by a taut thread held in the right position in front of the eye. From positions away from the centre of the sphere, on the other hand, the segments of great circles will as a rule look curved, and it will be impossible to bring a taut thread in coincidence with them.

Again, it is because Pozzo's nave ceiling is painted on a curved surface, namely a hemicylindrical one, that some of the straight parts of the architecture depicted in it look curved when viewed from the wrong position. This vast ceiling is so distant from the spectator that, when he is standing at the right place, he sees straight architectural features as straight even when he uses binocular vision. This would not be so for perspectives on curved surfaces placed close to the eyes. Then even if one eye is at the centre of projection, the other eye will be at a position significantly different from it. The surface itself will probably be seen as a curved surface, so that all lines in it may look curved, even if they can be covered by a taut thread, by virtue of the curvature of the surface to which they belong.

Parallel lines which are parallel to the projection plane

The central projections on a plane of a set of straight parallel lines which are parallel to the plane, are straight parallel lines. Thus, for instance, on a vertical plane of projection, all vertical lines are projected as verticals. In most of the pinhole photographs reproduced in this book, the camera was positioned with the help of a spirit level so that the photographic plate was in a vertical position. Vertical lines belonging to the architectural scene photographed are then all represented by vertical lines in the picture. (In Fig. 8.8b, on the other hand, where the camera was tilted by an angle of 20° from the vertical, the verticals in the architecture are no longer vertical in the photograph.)

Consider now the case of two horizontal lines situated in a vertical plane. Such a pair of parallels may be illustrated by the lines at the top and at the base of a long horizontal wall. In the section on Natural Perspective, it has been seen that the visual angle subtended by the height of the wall, for different positions along its length, is largest for that part of the wall which is nearest to the eye. This angle becomes smaller and smaller towards both ends of the wall: it would become smaller than any small value chosen at will if the length of the wall were supposed to be extended indefinitely. Again, it has also been seen in Chapter 6 that the curved retinal images of two such horizontal lines tend to converge in both directions. If we consider a plane of projection vertical and parallel to the wall, however, the central projection of the horizontal top and bottom lines of the wall is a pair of parallel, horizontal, straight lines.

This simple geometrical result has repeatedly proved a stumbling block. It has caused...
a considerable amount of controversy, and it has even led to various suggestions according to which straight lines should be depicted as curved on a plane surface of projection. Joosling in 1835 made the following comments:

The fact, however, has puzzled several; for many good draughtsmen are neither mathematicians nor geometricians; and they have asked, Will not the two horizontal and parallel right lines, for example, forming the top and base of a long wall, appear, to a person who may stand in front of the middle of it, to get nearer and nearer to each other the further they extend both to the right and to the left of the centre? Have not such lines, therefore, the appearance of curves? And ought not their representation, when drawn on a plane, as thus viewed, to be curved lines, as there is no appearance of angle in the middle?

The answer to these questions is, that straight parallel lines on the plane of projection, when viewed from the point of sight, will appear to approach each other; and also appear as much curved, in proportion to their length, as the long lines of the top and base of the wall would so appear from the point, at a proportional distance, from which they were viewed.

In other words, the height of the wall in positions more distant from the eye subtends smaller visual angles than in nearer positions—simply because of the increasing distance from the eye, the height of the wall being a constant. When the eye is at the centre of projection, the angles subtended by the height of the projection of the wall in corresponding positions become smaller, to exactly the same extent, for the same reason. This might be expressed by saying that, while the wall is seen foreshortened both towards the right and towards the left, the projection is also seen foreshortened in exactly the same manner—the word ‘foreshortened’ here referring to the visual angles of natural perspective, not to the shape of the projection. The projection of the object on the plane is itself an object, our vision of which obeys the laws of natural perspective. Whereas Joosling’s remark to some extent introduces the controversial notion of the subjective curvature of straight lines (an appearance which some claim to be aware of, whereas others deny it) the present problem can be solved purely as the objective basis of natural perspective and visual angles.

The perspective of the wall is not a replica of appearances or subjective impressions. It is not a replica of the retinal image. Neither is it a direct recording of the relevant visual angles.

The picture in perspective is a substitute for the actual objects, so constructed that, when the eye is at the centre of projection, the corresponding visual angles are always the same for the objects and all their component parts, on the one hand, and for the corresponding projection of these objects and all their parts, on the other. Under such conditions, the eye being at the centre of projection, both the top and the base lines of the wall, and their projections as two straight parallel lines on a transparent Leonardo window, form on the retinas images which are in coincidence with each other, superimposed point by point. The exact shape of the retinal images is irrelevant, as well as the fact that when the eye rotates in its orbit from one position to the other, the retinal images of the objects and of their projection shift onto different parts of the retinas; thereby they alter their shapes to some extent, but both in the same way so that they remain in coincidence with each other.

Parallel lines at an angle to the projection plane

In Fig. 8.6a the horizontal lines in the façade of the Arch are parallel to the vertical plane of projection. This corresponds to the case which has just been discussed. In Fig. 8.6b, taken from exactly the same centre of projection, these lines make an angle of 25° with the projection plane. Here all the horizontal lines of the façade are projected as straight lines which converge towards a point on the left of the picture, which is called their vanishing point. This is the point where an auxiliary line, parallel to the horizontals of the façade and passing through the centre of projection, pierces the plane of the photograph.

This follows from what has been said in the section on Natural Perspective in Chapter 5 with regard to sets of parallel lines considered together with the auxiliary parallel passing through the centre of rotation of the eye. As one considers positions on the parallels which are further and further removed from the eye, the visual angles subtended by the distances between the parallels become smaller and smaller. This applies to the distances of any one of the parallels to the auxiliary line. Now the perspective image of the whole of the auxiliary line is the single point where it pierces the plane of projection. Since the visual angles subtended by the distances of the projections of all the other parallels to the auxiliary line must, like the angles subtended by the distances between these parallels themselves, become smaller and smaller with increasing distance, these straight projected lines must necessarily converge towards the point image of the auxiliary line. Except when parallels are parallel to the projection plane, any given set of parallel straight lines will have its own vanishing point defined by the direction of the parallels, which determines where their auxiliary line intersects the plane of projection.

In both Figs. 8.6a and b the part of the actual Arch nearest to the centre of projection was close to the right inside vertical edge of the passage through the Arch; it was at a distance from the right end of the Arch equal to about one-third of the width of the whole Arch. This is the position where the height of the actual Arch subtended the
largest visual angle from the centre of projection. (This is so, of course, regardless of which projection surface may be chosen.) Now the preceding reasoning relating to Fig. 8.6b is clear as far as the parts of the horizontal lines situated on the left of the nearest position are concerned. There the angles subtended by the height of the building do decrease towards the left. From this nearest position towards the right, these visual angles must also begin to decrease, however. Yet the straight projections of the top and bottom lines of the Arch still diverge; they do not converge. The explanation is that the distance from the eye to the projection plane increases, but in such a fashion that the angles subtended by the projected height of the building do in fact decrease towards the right, even though the projected height does increase in size.

In such a case as that of Fig. 8.6a it has been seen that the angles subtended by the horizontal projections of the distance between the top and the base line do decrease both towards the right and towards the left, being largest at the position nearest to the eye. In Fig. 8.6b the angles vary in exactly the same manner, as indeed must be the case since, with the eye at the right position, that is at the centre of projection, both photographs when held in the right position would be exactly to cover each other and to cover the actual Arch.

The reason why there is no vanishing point on the right in Fig. 8.6b is that the surface of projection is a plane, so that the ancillary line intersects the picture in one point only. If a spherical surface of projection were used instead of a plane, however, with the centre of the sphere as centre of projection, then there would be for each set of parallels two vanishing points, at the two ends of the diameter of the sphere parallel to the set. The set of parallel straight lines would then be projected as great circles, and would meet at both vanishing points, in the same way as all meridians on a terrestrial globe meet at the North and at the South pole. In such a case, and in this case only, visual angles would be proportional to the corresponding segments of great circles they subtend on the surface of the sphere.

Importance of the orientation of the projection plane. For a plane surface of projection there is one vanishing point for all the parallels of a given set, when they are not parallel to the projection plane. The position of the vanishing point of a given set, for a fixed centre of projection, varies with the orientation of the projection plane. A third photograph of the Arch made from the same centre of projection on a vertical plane with the camera pointing towards the right, instead of the left as Fig. 8.6b, would show the same horizontal lines now converging towards the right instead of the left. A third, different projection of the same view from the same centre would thus be obtained. In the special case where the parallels are parallel to the projection plane, as in Fig. 8.6a, their projections are parallel in the picture.

These instances show that, all other things being equal, the pattern formed by the projections of sets of parallel lines is strongly dependent on the choice of the plane of projection. Strictly speaking perspectives such as Figs. 8.6a and b are not different aspects of the same view. They are the same view projected on different surfaces, the views and all its projections producing unreal images in coincidence for any given position of the eye in its (fixed) orbit. If we are aware of different aspects when we look at an object such as the Arch without changing our standpoint, this is so probably because we remember pictures in perspective of similar objects, and imagine how the Arch could be depicted on different planes.

Orthogonals to the plane of projection

Principal point. In the case of a set of straight lines which all are perpendicular to the plane of projection, the vanishing point is called the principal point of the perspective. It is of course the projection of the ancillary line perpendicular to the plane and passing through the eye, that is the centre of projection. The distance from the projection centre to the principal point is called the distance of the picture or of the projection plane; it is the shortest line joining the ‘point in the eye’ to the plane.

It should be stressed that the principal point is not a characteristic point relating to the eye or the vision of the spectator. In all this the eye is always free to move around its centre of rotation. For a given position of the centre of rotation of the eye, any number of projection planes can be chosen, and each will have its own principal point. In Fig. 8.6a the principal point is the vanishing point of the orthogonals to the façade which are seen within the passage through the Arch. In Fig. 8.6b these lines are no longer orthogonals to the plane of projection; their vanishing point is now to the right of the principal point relating to the new surface of projection.¹

Horizon. All the ancillary lines relating to sets of horizontal parallels are horizontal. As they all pass through the centre of projection, they define a horizontal plane which cuts the vertical plane of projection as a horizontal line, which is called the horizon. The horizon, so defined, therefore contains the vanishing points of all horizontal lines, including the principal point.

Frontal projections. Many paintings containing architectural elements have been made on a plane of projection parallel to the façade of the building represented, as in

¹ In the case of a spherical surface of projection, when the centre of the sphere is the centre of projection, there is no such 'principal' point. Because of the symmetry of the sphere, all straight lines passing through the centre intersect the surface of the sphere under normal incidence, and it is obvious that here there is only one possible 'orientation' of the surface of projection relative to the objects to be projected.
Fig. 8.6a. This is so in Piero della Francesca's Flagellation (Figs. 7.2 and 7.3). The orthogonal to the facade are then also orthogonal to the plane of projection, so that their vanishing point is the principal point of the perspective. In such cases the projection of the plane of the facade is geometrically similar to the facade itself. Objectively horizontal and vertical lines belonging to the facade are depicted as horizontal and vertical lines. A circle in the plane of the facade is projected as a circle.

Bearing in mind the example of Figs. 8.6a and b, it is important to realize that this is essentially due to the particular choice which has been made of the plane of projection. For the same projection centre, that is from the same position of the rotation centre of the eye, a vertical plane at an angle to the facade would change the pattern of lines considerably. The classical pattern of parallel horizontal lines in Fig. 7.2 would be entirely altered. Although the new picture would be another representation of the same view of the same scene, it would be a different painting. It could be made to appear the same as the original painting only by using the special arrangements of the experiment of Chapter 8, which do not correspond to the usual way of viewing paintings.

Parallel lines at 45° to the projection plane

Distance points. All sets of parallel lines at an angle of 45° to the plane of projection have a vanishing point whose distance from the principal point is equal to the distance of the latter point to the centre of projection. Consider those horizontal parallel lines at 45° to the projection plane, which are inclined to the left. Their vanishing point will be at $D_1$ in Fig. 10.1, where $O$ is the centre of projection, $LM$ the trace of the plane of projection, and $P$ the principal point. The angle $O D_1 P$ is equal to 45°, the line $O D_1$ being the auxiliary parallel of this set. Similarly horizontal parallels inclined to the right will have their vanishing point at $D_2$, the angle $O D_2 P$ being equal to 45°. The points $D_1$ and $D_2$ are called distance points. The principal point being at $P$, the angles $O D_1 P$ and $O D_2 P$ are right angles. Since the angles $O D_1 P$ and $O D_2 P$ are both equal to 45°, the angles $O D_2 P$ and $O D_1 P$ are also equal to 45°, so that the two triangles $O D_1 P$ and $O D_2 P$ are isosceles. Therefore $P D_1$ and $P D_2$ are both equal to $OP$, the distance of the projection centre to the plane of projection; and the distance $O P$ is of course equal to half the distance between $D_1$ and $D_2$.

Reconstruction of a scene depicted in perspective

Accordingly, in a picture in perspective containing a horizontal square which has one side parallel to the picture plane, the distance of the picture from the projection centre can in principle be determined by producing the diagonals of this square until they meet the horizon of the picture. These diagonals, being at 45° to the picture plane, have perspectives which intersect the horizon of the picture at the distance points. This is of great help in reconstructing the plan and elevation of the architecture depicted in a painting, as was done by Carter in Fig. 7.3 for the 'Flagellation' by Piero della Francesca. The principal point is the vanishing point of the orthogonal to the picture plane. Together the position of this point, and the distance, define the position of the projection centre chosen by the artist, that is, the position of his eye relative to the picture plane. In practice the problem is not always an easy one to solve, however, because it is usually difficult to produce the relevant lines with sufficient accuracy—even in the case of a photograph. It is advisable to use as many lines, or sets of lines, as possible to be able to check the results against one another.

All this may become clearer by referring to the photograph of Fig. 10.2, where the plane of projection was parallel to the balustrade represented. The distance points $D_1$ and $D_2$ have been obtained by producing the diagonals of the square bases of the balusters. The diagonals inclined towards the left all meet at $D_1$; those inclined towards the right all meet at $D_2$. The principal point $P$ was obtained by producing these sides of the square bases and tops of the balusters which are perpendicular to the picture plane. The three points $D_1$, $P$ and $D_2$ are on the horizon of the picture, that is on the trace of the horizontal plane passing through the centre of projection. Thus, the centre $O$, that is the pinhole of the camera, was on the line perpendicular to the picture at $P$, at a distance equal to $PD_1$ or $PD_2$. 

10.1 Distance points.

The plane of the paper is a horizontal plane containing the centre of projection $O$ and intersecting the vertical plane of projection along the horizon $LM$. The line $O P$ is perpendicular to the projection plane; its length is the distance of the centre to the plane. The two lines $O D_1$ and $O D_2$ form an angle of 45° with $O P$; consequently $D_1 P$ and $D_2 P$ are equal to $O P$. The points $D_1$ and $D_2$ are the distance points, that is, the vanishing points of the horizontal parallels at 45° to the projection plane. See Fig. 10.2.
The angle $D_O D$ was therefore equal to $90^\circ$. The large size of the angle subtended by this photograph accounts for the marginal 'distortions' which appear in it. It will be noted that Piero della Francesca's painting, Fig. 7.2, on the other hand, subdues an angle considerably smaller than $90^\circ$.

**Subjective curvatures**

Many observers find that when they direct their gaze to the side of a long straight line, for instance a searchlight beam in the night sky, the line appears curved to them, presenting its concavity towards the point to which their gaze is directed. The straight line looks straight when looked at directly, however. Thus the apparent curvature changes, or disappears altogether, according to the direction of the gaze. A straight horizontal line, as the gaze is moved progressively upwards, will in turn look concave downwards, straight, and then concave upwards. These subjective curvatures are apparent only in indirect, peripheral vision. This probably explains why some observers report that they cannot see them. Normally, of course, we look directly at any object which attracts our interest, in which case a straight line looks straight, so that in everyday life such subjective phenomena are rarely noticed.

When an enlargement of the pattern of Fig. 10.3 is viewed with one eye placed at the distance indicated by the length of the line $A$, the observer keeping his gaze fixed upon the centre of the pattern, the figure appears to many observers as an ordinary checkerboard made up of black and white squares equal in size and arranged in straight lines. The curvatures of the lines used in the construction of this pattern have been chosen so as to compensate for the subjective curvatures of straight lines which would be seen at the corresponding degrees of peripheral vision.

From the standpoint of linear perspective, it must first be pointed out that if the line of a searchlight beam, for instance, is covered by a taut thread, both beam and thread undergo the same changes of apparent curvature according to the direction of the observer's gaze. Consequently, to argue that an objectively straight line should be depicted in a perspective drawing as curved appears to be instance of the El Greco fallacy. Whatever the straight line looks like subjectively, there is *prima facie* evidence that a straight line depicting it in the drawing will have the same subjective appearance (whatever this might be) when the viewing conditions are the same. The subjective deformations of the one must be expected to be the same as the deformations of the other. This, however, assumes that we can examine both lines at leisure.

Now the case of the path of a shooting star or a meteor is an exception to this, for there is not time to move the eye to examine such a transient phenomenon. Hence it may well be justified to draw the straight path of a shooting star as a curve, if it appears curved
But the special subjective curvatures which are observed in binocular vision must be examined before turning to a discussion of the systems of curvilinear perspective which have lately been put forward.

Luneburg's theory of binocular vision. Luneburg (1947) and his followers (Blank, 1959) have put forward a mathematical theory of binocular visual space perception according to which visual space would be a non-Euclidean (hyperbolic) space. Luneburg's visual space is a mathematical concept introduced to account for the observer's subjective estimation of distances, either between observer and object, or between object and object. These subjective distances usually differ from the actual distances measured in physical space. This leads to particular illusions. The theory refers to binocular, stereoscopic vision, studied under special conditions, chosen precisely to eliminate clues which are not specific to binocular vision (Blank, 1959). The main experimental evidence is given by the alley experiments, to which Blumenfeld drew attention in 1913. In such experiments, the observer is as a rule asked to adjust in a dark room the positions of pairs of small light-sources so as to form an alley, either with sides which appear parallel to him, or in which the lamps of all the pairs appear to have equal separations. The main experimental difference is, however, that for the parallel alley the intermediate lights are left on, whereas for the distance alley they are put out. Objectively the alleys so obtained are found to have sides which are curved, and furthermore the two tasks lead to different alleys, the parallel alley lying inside the distance alley. It seems that most experiments so far have been made with all the lamps situated in the horizontal plane passing through the two eyes of the observer (Ogle, 1952). It is a rather old observation, on the other hand, that the parallel sides of a straight road, for instance, may appear curved to some observers (Fig. 10.4).

As far as the theory of linear perspective is concerned Luneburg's mathematical
theory (which refers to subjective observation, not to the physical space of optics) is less fundamental than the illusions it seeks to explain. It may be noted that psychologists such as Graham (1951) and Ogle (1962) feel that the theory may lack sufficient experimental foundation. The illusions it seeks to explain relate specifically to binocular vision. Now, while binocular vision enters into the perception of pictures with regard to the spectator's awareness of the picture surface, the illusion or suggestion of depth produced by a picture depends on the use of clues and elements which are not those of binocular vision. Under ordinary conditions of binocular vision the illusions occurring under special dark-room conditions often fail to take place, no doubt because many other perceptual factors then come into play besides those of 'pure' binocular vision. Even in the dark-room experiments made on the perception of parallel lines by ten Doesschate and Kylström (1956), in which the 'lines' were luminous straight tubes, no subjective curvature of the lines was observed.

The main fact is that in ordinary vision, for most observers, there are no obvious subjective curvatures which are present only when both eyes are used, and which disappear when one shuts one of the eyes. (The curvatures discussed at the beginning of this section are seen with one as well as with two eyes.) None the less, it is advisable to keep an open mind with regard to subtle differences between binocular and unocular vision, which may explain certain special features in the works of some representational painters.

Systems of 'curvilinear perspective'. Various systems of perspective have been proposed, by Haack (1875) and Panofsky (1927) among a number of others, according to which the representation on a plane of objectively straight lines should, for most of the lines, be curved. Now according to the argument developed in the present book the Optics of Euclid does not lead to such a conclusion. The theory of central perspective which has been discussed rests upon the principles contained in Euclid's Optics and Elements—whereas, contrary to what Panofsky argued, are definitely not in disagreement with the perspective theory of the Renaissance. (The 'curvature of space' of the theory of relativistic, which is sometimes appealed to, is of course quite irrelevant.)

It would appear that it is the fact that the retina, and perforce the retinal image, are curved, which leads some authors to the idea that a truly 'physiological' perspective should consist of some kind of pseudo-development upon the picture plane of an image curved in shape like the retinal image, which allegedly would lead to systems of 'curvilinear perspective'. But, first, the retinal image is not what we see: what we see is the external world. Secondly, the geometrical construction of such a pseudo-development remains obscure—unless it leads back to central, rectilinear, perspective. It would be pointless to reiterate the argument that central perspective, in which straight lines are never projected as curves on a plane, is the only method which is capable of producing a retinal image having the same shape as the retinal image of the actual objects depicted.

The fact remains that a certain number of paintings, discussed for instance by White (1957), contain curved lines which apparently are meant to represent straight lines in the objects depicted. First it may be pointed out that in the world around us relatively few objects have outlines which are exactly straight. And exactly straight lines are often found displeasing. Artists therefore may have copied, sometimes with some exaggeration of the curvatures, lines which were not exactly straight; or they may even have introduced non-existent curvatures for aesthetic purposes. Secondly the composition of some of the paintings may have been influenced by the appearance of the reflection given by a convex mirror, where most straight lines look curved (see Schwarz, 1959). This mode of depiction has the advantages of squeezing into a small compass the peripheral parts of the scene represented (rather like 'fish-eye' photograph cameras which cover a very wide angle) but at the cost of not giving an exact central projection.

The possibility can by no means be ruled out, however, that some of these curvatures may have their origin in the fact that the artist may have noticed subjective curvatures due to the use of binocular vision, and introduced them in his work. Painting and drawing although essentially unocular modes of representation, may thus have been made to reproduce effects belonging specifically to binocular vision. This, however, is a possibility which seems to have been little studied.

Curvatures in Doric temples. Many of the ancient Greek temples in the Doric style, for instance the Parthenon at Athens and the so-called Temple of Neptune at Paestum (both of the fifth century B.C.) show subtleties of curvature and inclination, commonly called 'refinements', which greatly contribute to the impressive harmony of these masterpieces of architecture. The stylobate, that is, the main floor of the temple, is slightly curved, forming a convex surface. The columns also have a convex profile.

1 'Curvilinear' systems of perspective have been discussed and criticized for instance by Gioselli (1937, 1937-8, 1966), ten Doesschate (1964) and Zanetti (1951).

2 A book by Barre and Flouon (1968) again advocates the use of 'curvilinear perspective', and is illustrated by many drawings based on the authors' method. This rests on a peculiar way of using an axiom put forward by Bouligand, Flouon and Barre (1964) according to which 'The same magnitude appears the smaller as it is more distant from the observer'. Now this axiom is in full agreement with Euclid, if magnitude is taken to mean visual angle. But, for the present writer, the way the authors apply it for instance to the depiction of a long wall seems impossible to understand. For, according to the authors, if the plane of the projection were in coincidence with the plane of the wall, the wall should be depicted, 'full size', as two curved lines converging at each end. So the picture of the wall, full size, would utterly differ from the wall itself.

3 As Mr R. A. R. Carrer pointed out to me, if the artist 'measures' with a pencil or ruler held at arm's length, and if he simply transfers these measurements onto his canvas, he will obtain curved representations of straight lines. In some cases such representations may thus be due merely to the use of this procedure.
(entasis), strongly marked at Paestum, rather less so in the Parthenon. Some of these curvatures probably played a practical role: the convexity of the stylobate is useful for drainage. Yet these refinements must also have had 'beauty' as their object (Dinsmoor, 1950; Robertson, 1964). In the Parthenon the columns at the corners are thicker than the others; it seems clear that the purpose of this was to avoid their looking too thin against the bright sky, since a bright background seems to 'eat' into the outline of dark objects in front of it. Apart from this, it is hard to give any convincing explanation of the refinements, at any rate of the curvatures, on the basis of optical facts or theories—even though the Greek are said to have had such theories. It is sometimes argued that these objective curvatures were used to compensate for effects of subjective curvature. But there is no reason to believe that the architects did wish to make the relevant lines look quite straight in any case. Consideration of these curvatures of the Doric style mainly suggests the rather obvious conclusion that slightly curved outlines, when suitably chosen, can help to produce a most extraordinary artistic impression. Choisy (1904) and Giossoli (1966b) have examined in more detail possible optical explanations of the Doric refinements.1

These curvatures have been mentioned here largely because they often are discussed in connection with the theory of linear perspective. The link between the two subjects seems tenuous, however, since perspective deals with the problem of the accurate representation of given objects, whereas architecture is not a representational art.

On a further visit to Paestum it was pointed out to me by my wife—as Mr A. S. Event had done on a former occasion—that when standing in front of the narrow end of the temple of Neptune, at a distance such that the building subtended some 3° at the eye, then both the stylobate and the 'steps' looked rather strongly curved, this apparent convexity being upwards; that is, in the same direction as their slight objective convexity. This objective curvature therefore cannot have been intended to compensate for subjective curvatures: it only reinforces them. Furthermore Mr S. A. Medd pointed out that objectively straight steps, as those of S. Maria Maggiore in Rome, also look markedly convex upwards. No simple optical principle seems able to account for these subjective effects—which, it may perhaps be stressed, were noticed by several persons independently of one another.

How pictures look when viewed from the wrong position

Influence of preconceived ideas

Theoretical ambiguity of perspective. Any given picture in perspective seen with one eye placed at its centre of projection corresponds to an infinite number of possibilities, for the central projection of an object on a surface is also the projection of any number of other possible objects—which, for the projection centre considered, would all give the same projection on the same surface. Thus the well-known rooms built by Ames have shapes which are central projections of ordinary rooms, but are in fact completely distorted in comparison with the rooms we are used to in daily life. Ames’s rooms look like ordinary rooms. Human figures in them, however, appear much too small or much too large, according to the position they occupy, because the figures appear in a general perceptual framework which has been accepted as a room of familiar shape (Ittelson, 1952).

The manner in which the scene depicted in a picture is seen also depends very strongly on the spectator’s preconceived ideas; these ideas becoming reinforced into a whole framework of consistent relationships in the case of a whole scene consisting of objects with which the spectator is familiar—especially when light, shade and colour have been skilfully used by the artist.

In theory, a picture in perspective might be expected to be thoroughly ambiguous, even when viewed from the right position. In practice, most pictures representing scenes made up of elements of a kind with which the spectator is familiar are far from being ambiguous. In the case of Pozzo’s nave ceiling, all spectators, when at the right position, see the painted architecture as a continuation of the real architecture of the church, and when they are at the wrong position they see it undergo the same strange deformations. This absence of ambiguity must be explained by the spectator’s preconceptions concerning the objects and figures depicted.
Perspective illusions in architecture. Perspective illusions in architecture also rely on the spectator's preconceived ideas concerning the building he sees. Thus the arcade of Fig. 11.14 looks, in reality as in the photograph, like a long arcade of constant width and height. Yet it was built by Borromini (1599–1667) on the plan of Fig. 11.1c. The actual arcade is the solid perspective, in three dimensions, of an imaginary arcade of normal shape. It is built so that each of its parts, each of its columns, for instance, would cover accurately the relevant part of the imaginary arcade of constant width and height, for a spectator placed at the right position at the entrance of the arcade. From this position, the arcade looks much longer than it is, the decrease in its actual height and width with increasing distance from the spectator is not noticed, and the statue at the end of it looks much larger than its real size (Fig. 11.1d). A man standing at the end of the arcade looks like a giant, an effect similar to that produced in Ames's rooms. Conversely, viewed from the end opposite to the entrance in the other direction, the arcade appears much shorter than it does from its entrance (Fig. 11.1b).

A more subtle effect has been realized by Michelangelo (1475–1564) in his design of the Piazza of the Capitol at Rome. As shown in the plan of Fig. 11.2c the façades of the two palaces on either side of the Piazza are not parallel. The space separating them is larger at the back of the Piazza, near the Palazzo dei Senatori, than near its entrance, at the top of the ramp which leads up to the Piazza. The result is that, when the spectator stands near the Palazzo dei Senatori, the Piazza gives the impression of being deeper than it is (Fig. 11.2b) because he tends to take it for granted that the buildings are arranged on a rectangular plan (Fig. 11.2c). The actual convergence of the actual façades is mistaken for the increased perspective convergence which would be given by longer parallel façades. The design of the pavement of the Piazza contains no straight lines, which might counteract this effect, but consists of intercrossing curved lines which rather help the illusion.

To the spectator entering the Piazza from the top of the ramp, on the other hand, the Piazza appears less deep than it is (Figs. 11.2a and 11.2d) and looks rather square in shape. It seems possible that this indeed was the main effect that Michelangelo intended to achieve on this historical site, where limitations of space curtailed his freedom.

Neville's Court in Trinity College, Cambridge, is built on a plan similar to that of the Capitol Piazza. It was completed by Sir Christopher Wren (1632–1723), who built the College Library which closes the far end of the Court. From the opposite side, which gives an imposing view of Wren's Library, most spectators agree that the Court looks square. The spectator's position is then similar to position 5 in Fig. 11.2d. Now Wren was of course influenced by Italian architecture: it seems possible that he was inspired by the arrangement of the famous Capitol Piazza at Rome and chose the position of his Library, relative to the existing three sides of the Court, so as to achieve the same effect as Michelangelo.
Theoretical deformation of pictures viewed from the wrong position

Returning to the case of flat pictures in perspective, drawing 98 of Fig. 11.3 is the perspective of an arcade, made from the centre of projection marked by the point O in drawing 97 which gives the plan of the arcade. Exactly the same central projection (drawing 98) would be obtained for the arcades the plans of which are given in drawings 95 and 96, using the respective projection centres marked O on these plans. The projection of the picture on the ground plane is the line ab in each of the three plans. The

Plan of the Borromini arcade in the Palazzo Spada
(From Lecchi (1846–57), Edifici di Roma Moderne.)

Diagram of the perspective effect produced by the arcade in the Palazzo Spada.
On the implicit assumption that the sides AC and BD are parallel, as they would be in an ordinary arcade, the spectator S at the entrance thinks he sees a longer arcade the sides of which are AC and BD.

Plakatsche photograph of the Piazza taken (a) from the top of the ramp leading to it, and (b) from a position near the Palazzo dei Senatori, which is visible in the background of (c). The plan of the Piazza is given in Fig. 11.2c. The diagrams of Fig. 11.2d and e show why the Piazza looks deeper from the second than from the first position.
III. 36—37

Theoretical Deformation of Pictures

Diagrams of the perspective effects of the Piazza of the Capitol

In III. 36 the spectator $S_1$ at the top of the ramp sees the façades $AC$ and $BD$ as $AC'$ and $BD'$.

In III. 37 the spectator $S_2$ near the Palazzo dei Senatori sees these façades as $AC$ and $BD$ and the Piazza appears deeper than it is, whereas in III. 36, from $S_1$ it appears shorter.

centre of projection always remains at the height above the floor, indicated by the line of the horizon passing through the point $P$ in drawing 98. Accordingly, if the perspective of drawing 98 is viewed with the eye placed relative to it not at the point $O$ of drawing 97, but at the point $O$ of drawings 95 or 96, the perspective now should appear to represent one of the oblique arcades of drawings 95 or 96. Thus when the eye moves to the left of the right position, the arcade should appear oblique towards the right. When the eye moves at the same time to a position more distant from the drawing, the arcade should also appear to become more elongated, as in drawing 95.

For most observers deformations of this kind are neither obvious nor striking. Many people only notice such deformations when their attention has been drawn to them, and then only after some practice, even if they use one eye only. The apparent deepening of the view when the eye moves farther from the picture, and its apparent flattening when it moves nearer, are the deformations least difficult to detect. No doubt, besides the spectator's subsidiary awareness of the picture surface, the spectator's preconceived idea that the perspective cannot be meant to represent arcades with such unusual shapes as
Theoretical effect of displacements of the eye on the appearance of a perspective

Drawing 98 refers to the arcade the plan of which is given in 97, for the eye position marked O in 97. It is also the perspective of the arcades the plans of which are given in 95 and 96, when the eye's position is moved to the corresponding positions marked O in 95 and 96. Theoretically, therefore, the drawing should appear to represent arcades 95 or 96 when the eye is displaced from the position O in 97 to the relevant positions for 95 or 96. In practice, for most observers, such alterations are rather difficult to notice. (From La Gournic (1859), Traité de Perspectives Lithaire.)
those of drawings 95 and 96 hinders their seeing the oblique deformations. (How inconspicuous these deformations are is shown by comparison with the deformations shown by anaglyphs (pages 163–4) in which the scene is really seen in 3 D and the surface of the pictures is ‘invisible’.)

**Stable elements in pictures.** Taking it for granted that the scene represented is of a kind with which the spectator is familiar, the theory of perspective predicts that some elements of the picture must remain unchanged when the spectator moves away from the right position. Thus a straight line foreshortened to one point in the picture seems directed towards the eye of the spectator, wherever he may stand. Each and every spectator sees it directed towards himself. This is so, first, because only lines of sight, passing through the spectator’s eye, are represented in this way, and, secondly, because this representation does of course remain unchanged in the picture when the spectator changes his position. Accordingly, a rifle depicted so that its muzzle hides entirely the barrel will appear to threaten the head of every spectator. If part of the barrel is depicted above the muzzle, in strong foreshortening, every spectator will see the rifle pointing towards his body.

The sighting eye of the rifleman will also appear to look at the spectator, wherever he may be. The apparent direction in which the rifleman seems to be looking depends upon the depiction of the various parts of his eye. In spite of the effects of natural perspective, the drawing of the eye hardly changes its aspect when one moves relatively to it, whereas the aspect of an actual eye would definitely be altered. Thus if, from one position, the eye of a real person looking steadily in one direction shows, say, the pupil in between two equal triangular areas of the white of the eye, then from another position the two areas of white may become unequal, and one of them may even become invisible. As no such change occurs in the eye depicted on a surface, a portrait either ‘looks’ at all the spectators, or ‘looks’ at none of them.

What has just been said applies to perspectives on curved as well as on plane surfaces, and to vision with two as well as one eye. Again it is valid for ordinary pictures and for special cases such as Pozzo’s painted ceiling.

In the case of a picture in central projection on a plane, all objectively straight lines always remain straight lines wherever the spectator’s position—except of course for the lines of sight, projected as single points. If the plane of projection is vertical, vertical lines are verticals in the picture. Consequently a building will appear to stand upright even when its picture is viewed from the wrong position.

**Relatively stable elements under special conditions.** If now the eye of the spectator remains in the plane of the horizon of the plane picture, that is in the horizontal plane passing through the centre of projection and the principal point of the perspective, then from a position away from the projection centre, the depiction of lines which are objectively horizontal and parallel will still give the impression of horizontal and parallel lines—because their vanishing point is on the horizon. It can also be shown that the ratios of the parts into which a horizontal line is divided appear to remain unchanged under those conditions. Thus if the pediment of a building depicted in perspective is an isosceles triangle, it will continue to appear as a triangle having two equal sides; similarly a gothic arch will still appear symmetrical relative to its vertical axis. Again the disposition of the windows in a façade in perspective remains unchanged when the eye is at the wrong position, provided always it remains within the plane of the horizon of the picture (La Gournerie (1859) p. 223). If, say, a painting is hung at the right height in a gallery, this condition will be fulfilled, at least approximately. (If a painting has to be placed too high on the wall, it can be inclined forward, so that the eye of the spectator still can come within the plane of its horizon.)

When the eye moves out of the plane of the horizon, the deformation of the picture should become much more important, but La Gournerie points out that even then the deformation will occur in a regular manner if the picture is accurately drawn in perspective. On a flat surface the composition thus retains a considerable degree of harmony.

It must also be noted that the arrangement of cast shadows depicted in perspective is not upset when the eye views the picture from a wrong position.

**Ordinary pictures viewed with both eyes.**

The preceding paragraphs summarize some of the conclusions reached by La Gournerie (1859) who made a special study of the deformations which theoretically should occur in the appearance of a perspective on a plane when it is viewed with one eye from a position other than the projection centre. (The general discussion of the problem is related to the theory of mathematical homology.)

It is interesting to note that on this basis, in the case of objects bounded by straight lines, there are important elements in pictures in perspective which remain stable, even though La Gournerie’s discussion does not take into account the spectator’s subsidiary awareness of the picture surface in the perception of ordinary pictures. It is the case of objects having curved surfaces of known shape, discussed in Chapter 9, which shows most

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1 On the other hand, La Gournerie (1859) wrote that he was able to detect how accurately a picture was drawn in perspective by the fact that the deformations which occur when the eye is in the wrong position become more conspicuous when the perspective is correct.

Conversely, therefore, it may perhaps be surmised that paintings such as Cézanne’s, which are not drawn in exact central perspective, are able better to retain their own peculiar harmony for all positions of the spectator, precisely because they are not drawn in exact perspective.
definitely the complex and considerable role played by this awareness, especially when, as is normally the case, binocular vision is being used. It is clear that this must apply to the picture as a whole, including both straight and curved lines and surfaces. In general, therefore, the fact that many find it difficult to see the deformations theoretically predicted for a spectator who is not at the correct position, must be explained by an intuitive process of psychological compensation which is based both on the spectator’s awareness of the surface of the picture, and on his preconceived ideas regarding the components of the scene presented.

It is the existence of these processes which must largely explain that pictures in perspective can be used as widely as they are as representations of complicated scenes or objects, even for purely practical purposes. On theoretical grounds alone, and on the basis of our present knowledge of vision, it might have been thought that the only pictures which could be of real use would be stereoscopic pictures, or at any rate pictures set up in a special way to be viewed through a peep-hole. It has of course always been taken for granted, on the basis of experience, that this is not so. Perspective views are found to be most useful for instance to engineers and architects. Again, pictorial advertisements seen from a moving staircase (as commonly displayed, for instance, in the London Underground) do not, while one is carried by the side of them, show the strange deformations which might theoretically be expected to occur. As this work has attempted to show, however, the reason why this is so is not an obvious one.

Whereas in some very special cases, like Pozzo’s ceiling, the illusory space of the picture is indistinguishable from the physical space of daily life (as is also the case of Borromini’s architectural illusion) one is driven to the conclusion that this cannot be so for ordinary pictures. Such pictures produce an illusion of a very particular kind, to which we become accustomed as part of our education. This probably explains why it has often been reported that when a photograph is shown to people in whose civilization photographs and similar pictures in perspective are unknown, they fail to perceive what the photograph means to represent, at least until it has been explained to them.1

1 This is largely based on anecdotes, most of which seem hard to trace. In a private communication, however, Dr. A. W. Essel (formerly Deputy Keeper of the Botany Department, of the British Museum (National History)) told me that he found that members of the Chokwe tribe in Lunda in North-East Angola 30 years ago (in 1936) failed to understand the meaning of photographic pictures. But, as Dr. Essel notes, one cannot exclude the possibility that they might have regarded the photos as “unlucky” and refused to become involved. Segall, Campbell and Hendriks (1956) discuss this matter, and give a first-hand report of a Bush Negro woman who, when shown a photograph of her own son, was able to perceive the subject only after the details of the photograph had been pointed out to her.

Yet some primitive people, at any rate, while very skilled in the use of their eyes, are subject to the same perspective illusions as civilized people, even in the case of an object familiar to them. Allport and Pettigrew (1957) made visual tests on Zulu children in Natal, South Africa, using the rotating trapezoid window illusion described in 1951 by A. Ames, Jr. The illusion

A plane mirror—or, better, a system consisting of two such mirrors—does, on the other hand, give a complete illusion of ordinary physical space. The mirror, apart from a general alteration of the direction, sends to the spectator’s eye the very same light which would be sent by the real objects seen reflected in it. This applies to both eyes, and remains true when the spectator alters his position. What we see in the mirror in fact is reality itself. The plane mirror, when its surface is invisible, is the only optical instrument which functions in this way. (On these grounds, some even deny that the word ‘illusion’ should be used to describe the effect produced by a mirror.) In any case, the illusion produced by the stereoscope, trompe-l’oeil pictures, and ordinary paintings belong to other categories, since their aim is the re-presentation of reality.

Deformation in anaglyphs. The stereoscopic devices known as anaglyphs readily show the deformations predicted by the theory of perspective. Two central projections of the same scene or object are made from centres separated by a distance equal to the distance between the pupils of the two eyes. These two projections are drawn or printed on top of each other, in the same way as if the projections of a scene viewed through a Leonardo window were drawn in superimposition on the window, for each eye in turn—without moving the head. A different colour is used for the two projections. They are viewed through glasses having different colours for the two eyes, chosen so that the right eye only sees the projection corresponding to it, and the left eye only the projection corresponding to it. The object represented is then seen in three dimensions.

Now whereas in the case of an ordinary stereoscope the position of the eyes is fixed, in the case of anaglyphs the spectator wearing the colour filters can move about relative to the double picture. It is then found that the object seen undergoes strong deformations, of the kind forecast for instance for the arcade of Fig. 13.3 when the eye moves away from the centre of projection. In the case of anaglyphs, the observer is hardly aware of the is that instead of steadily rotating, as it actually does, the window is seen to sway back and forth in an arc of 90-180 degrees. The trapezoidal window is in fact the perspective projection of an ordinary rectangular window, and can be mistaken for such a window. Primitive Zulus, however, live in round huts devoid of windows and see hardly any rectangular objects; but they must be used to the sense of the vertical and the horizontal.

The authors report as their most striking finding that under conditions optimally chosen to produce the illusion (unilaterally at 20°) virtually as many primitive Zulus report the trapezoidal illusion as do urban Zulus or “Europeans”. The illusion really is one of binocular perspective, so that under particular sub-optimal conditions, when binocular vision is used, the illusion may not occur. The window is then seen to rotate as it actually does because the true information mediated by stereoscopic vision supersedes the illusion caused by perspective. Under these conditions the authors found that experience with, and identification with, western culture make it more likely for the illusion to occur: the subjects then are familiar with windows. In this particular case, therefore, Allport and Pettigrew write, “one may say that the primitive children see things “as they are” more often than do the children of civilization.”

17-3
surface of the picture, as a surface. Again, the sense of the third dimension given by the discrepancies between the two projections overcomes the possible effect of preconceived notions concerning the object represented. The deformations occur because, when the observer's eyes are in the wrong place, they are in fact presented with the two projections which would be given, for the positions of his eyes, by the deformed object which he sees. In the case of such stereoscopic devices, the theoretical ambiguity of a picture in perspective no longer exists, since there are now two projections of the same object made from two different centres. Observation of such deformations, shows, by contrast, how weak are the similar deformations in the case of ordinary pictures.

12 Imitation in painting

Imitation or representation evidently is not, in itself, the aim of representational art. It is only one of the artist's means of expression, like the composition of the subject of his painting, and of course the choice of the point of view and of the projection surface on which the painting is made. None the less, some painters are generally included among the most original artists, who precisely seem to have sought to imitate nature as closely as possible: the brothers Van Eyck, Leonardo da Vinci, Vermeer, Seurat.

Leonardo's writings on painting, for instance, show explicitly that he strove to represent what he saw. The question then arises: how can such an attitude be compatible with the creation of original work?

Nowadays this problem may appear to be an insoluble riddle, unless the question be brushed aside as utterly simple-minded. For the belief has become widespread that an exact, complete and objective representation of the visible world could be made by an artist, but that this 'mere imitation' would not be Art. Furthermore, it is believed that such a representation can be obtained 'scientifically' by the use of photography. Now, of course, if this were really so, the artist's task could only be a task of selection and transformation, which in effect would use as a basis and starting point a perfect objective representation, itself devoid of artistic interest.

But, in fact, such a perfect, objective, representation can be obtained neither in painting nor in photography. So, the striving of an artist after the (unattainable) ideal of 'copying nature' does not necessarily entail any loss of originality.

It may be noticed that, in connection with the art of acting, the fallacy concerning 'mere imitation' probably would not arise. It seems likely that most theatre-goers, even today, would disagree with Partridge's opinion of such an actor as Garrick. In his novel Tom Jones, Fielding (1707-54) tells us how Tom Jones took his friend Partridge to the theatre to see Garrick in the role of Hamlet. Tom Jones 'expected to enjoy much entertainment in the criticisms of Partridge, from whom he expected the simple dictates of nature, unimproved indeed, but likewise unadulterated by art'... At the end of the play, Jones asked him, "Which of the players he liked best?" To this he answered, with some appearance of indignation at the question, "The king, without doubt." Indeed,
Mr Partridge”, says Mrs Miller, “you are not of the same opinion with the town; for they are all agreed that Hamlet is acted by the best player who ever was on the stage”. “He the best player!” cries Partridge, with a contemptuous sneer, “why, I could act as well as he myself. I am sure, if I had seen a ghost, I should have looked in the very same manner, and done just as he did. And then, to be sure, in that scene, as you called it, between him and his mother, where you told me he acted so fine, why, lord help me, any man, that is, any good man, that had such a mother, would have done exactly the same. I know you are only joking with me; but indeed, madam, though I was never at a play in London, yet I have seen acting before in the country; and the king for my money: he speaks all his words distinctly, half as loud again as the other—Anybody may see he is an actor.1

Limitations of pictorial representation

Leaving aside such exceptional cases as Pozzo's ceiling, peep-shows, and ordinary trompe-l'œil paintings (in which invariably the subject matter has very little extension in depth), ordinary pictures viewed binocularly in the usual manner do not give a genuine three-dimensional representation of their subject matter. This point has been discussed at length in previous chapters. In spite of the fact that it may give a very strong suggestion of depth, the representation given by ordinary pictures is of a sui generis nature, and depends on the spectator's subsidiary awareness of the characteristics of the picture surface. This at once shows that ordinary photographs do not, any more than paintings, give a perfect imitation of the scene they represent.

Furthermore, it is more difficult for the photographer than for the painter to compensate for some other limitations of his technique, precisely because of the partly automatic nature of photography. The range of luminances in a photographic print or in a painting is often much more limited than the range actually existing in nature, as for instance in the case of an interior inside which the sunshine penetrates by small windows, through which the sky is also visible. Such scenes, which have been most successfully painted by Dutch artists of the seventeenth century, are notoriously difficult to photograph.

Again the general level of luminance may be much higher, for instance in a sunny landscape, or much lower, for instance in a moonlit scene, in reality than in the picture hanging in a gallery. Now visual acuity, colour, contrast, do vary with this level of luminance. The painter can compensate in part for such differences of luminance between his canvas and the actual scene he wishes to represent. Seurat's pointilliste technique imitates the effect of shimmering sunlight, even though the physical luminance of his canvas is much lower than that of the sunlit scene.1

1 The main principles of Seurat's technique, known as 'pointillisme', 'divisionnisme' or 'neo-impressionism', had been expounded by Brücke in 1877 and 1878 before Seurat painted his

Helmholtz (1871-3) and Brücke (1877) have discussed in detail such limitations of pictorial representation and the techniques used by artists to circumvent them. Far from giving a scientific recipe for making perfect pictorial imitations, their writings do in effect prove that in many cases it is impossible to achieve a perfect pictorial imitation with regard to light, shade and colour. Thus the 'utmost degree of perfection' postulated by Brook Taylor in his discussion of perspective is impossible to reach. It is for this reason that Pozzo's ceiling, even though it gives a real three-dimensional impression, does not produce a total illusion. Stereoscopic pictures similarly do give a 3D impression, but not a complete illusion; indeed the almost perfect impression of relief given by the stereoscope tends rather to emphasize, by contrast, the limitations of the photographic representation in other respects.

The case of moonlit scenes may provide a simple instance of the dependence of visual physiological effects upon the general luminance level. In recent years it has become well-established that Colour vision is mediated only by the cone receptors of the retina, whereas the rod receptors only mediate colourlessness, or at any rate monochrome, vision. Now in a dark moonless night the light is too weak to stimulate the cones—the centre of the fovea, which contains cones only, then is blind. The rods alone are functioning, and colours cannot be distinguished; only differences of light and shade are perceived. In moonlight cones are functioning to a certain extent, as well as the rods, so that some colours can be recognized. Now as moonlight is roughly one million times dimmer than sunlight, a man wearing light-tight very dark goggles with a transmission only about one part in one million will see (after he has become adapted to this low level of illumination) the sunlit scene around him as a moonlit scene, with pale colours, dark shadows, and blurred outlines.

Of course an artist would directly paint a moonlit scene with pale colours, dark shadows and blurred outlines, but his canvas is meant to be viewed at a luminance level much higher than that of the actual scene. The canvas then sends to the eye much more light than an actual moonlit scene, yet it does at least strongly suggest moonlight. It would be theoretically possible to paint a moonlit scene in bright colours and arrange to see it only in very dim light of the correct intensity, but, needless to say, such experiments are foreign to the art of painting.

The proof that the physical basis of colour, that is, the varying spectral composition

main pointilliste works. Brücke traced the elements of this technique back to Rubens and Murillo, discussing also textiles and mosaic. His book appeared in 1878 in a French translation, published together with a translation of Helmholtz's (1871-3) Lectures on Optics and Painting. It is most probable that Seurat read this book (which is still to be found in the Central Public Libraries). Brücke (1878) explains Seurat's aim and method more clearly than most later writings dealing specifically with Neo-impressionism; for instance the book by the painter Signac (1899), Seurat's discipline, fails to explain all this adequately.
of the light emitted or reflected by objects, is present at night as well as in daytime, is given by the fact that, using long exposures, colour photographs can be made of objects the colour of which cannot be seen because the light they send to the eye is too weak. Thus brightly coloured photographs have been made of nebulae which are so dim that their colours cannot be seen even with the help of the most powerful telescopes (see Firenze and Marriott, 1962).

The example of moonlit scenes shows that artists can readily compensate for the difference in illumination between their canvas and the scene they wish to depict. This is much more difficult in the case of photography. An ordinary colour film used to photograph a moonlit scene will produce a picture if the exposure is made long enough; but this brightly-coloured photograph will not correspond to what is seen.

Yet the idea that photography automatically gives an exact and complete representation of the world we see around us seems to have become embedded in people's mind at an early stage. Töpffer in 1865 was already worried by 'Daguerre's machine', which he seemed to have put on the same footing as a mirror. The 'Artists' Almanack' published by Punch in 1849 contains a remark which must have been current at the time: 'That he who only paints what he sees degrades himself into a daguerreotype'. In principle, it should have been sufficient to compare with reality (or with what we see in a mirror) the results given by photography to realize its deficiencies. The scientific prestige of photography must have prevented this conclusion from being drawn, a preconceived idea thus overruling the testimony of the sense of sight. This is an instance of how difficult it is to see.

Some psychological problems of the artist

Our visual perception is to a large part based on elements of which we are hardly aware and is influenced by previous experience and by preconceived notions of all kinds. A great part of some scientific studies necessarily consists in learning to see. Mrs Johnson Abercrombie (1960) has made a special study of this problem. She writes that students know how difficult it is to look with profit at, say, a skull, or down the microscope at a section of skin, until they have had some kind of instruction in what to look for and preferably have a diagram beside them. But on the other hand, teachers know that if students have such aids to observation they... tend to see what they expect to see whether it is there or not. Many a student's drawing resembles a textbook figure more than the specimen in front of him; and the more complicated the specimen is or the more unfamiliar, the more its picture looks like the book. How to tell students what to look for without telling them what to see is the dilemma of teaching.

1 « Les arts "d'imitation" se manifestent entre l'idée bien définie de représenter "exactement" la "réalité", de chercher à égaler ce qu'on obtient d'une bonne photographie ou d'un montage, et les effets de l'intervention du système vivant qui doit exécuter cette représentation. (Valéry, 1939.)

Two illustrations may be of interest here. In the engraving by Hogarth (1697-1764) part of which is reproduced in Fig. 12.1, the bottle on the right shows a bright reflection of a window in spite of the fact that in the room where this 'Modern Midnight Conversation'

12.1 Detail from Hogarth's engraving 'A Modern Midnight Conversation'
Note the reflections on the two bottles.

was held the windows must have been quite dark. Furthermore the reflection of the window is at the wrong angle, for the vertical sides of the window should remain vertical even when the bottle is tilted. Again, in another part of the same engraving, not reproduced here, the flame of a candle held almost horizontally is depicted as burning almost
in line with the axis of the candle, instead of rising vertically. There is little doubt that lapses of this kind (which are not uncommon in realistic paintings or drawings) are based on preconceived ideas of how things should be, instead of on direct observation of how they are.\(^1\)

Perhaps a more interesting case is given by Fig. 12.7 in which Huygens in 1659 reproduced the drawings of the planet Saturn made by earlier astronomers, including Galileo in 1610. Saturn had faced astronomers with a puzzle until Huygens, having built a better telescope than his predecessors, saw Saturn's ring and was able to explain the phases of this planet (Figs. 12.3 and 12.4). The drawings in Fig. 12.3 were made on the basis of

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\(^1\) Anyone who attempts to draw accurately will have noticed in his drawings errors which were caused by false preconceptions, especially when vision was not quite distinct. Thus, to take a minor instance, in a landscape made in the Balearic Islands, I drew the silhouette of a distant isolated chapel, built on the top of a smooth bare hill, as a rectangular mass flanked by a large square-topped tower. In fact, this chapel simply consists of a large rectangular hall, but there is next to it a clump of very large trees. It is this round clump which I mistook for a square tower—having also underestimated the actual size on account of the distance.

Euclid said that a rectangular object at a distance looks round. Five centuries later Sextus Empiricus (A.D. c. 200) referred with some insistence to this illusion: 'The same tower appears round from a distance, but square from close at hand' (Outlines of Pyrrhonism, Bk I, § 35, 118, Bk II, 555). What this sceptical philosopher apparently failed to notice, however, is that round objects may also look square.

The results of some dark-room experiments have a bearing on this. A large luminous circle, or half this circle, was flashed on a screen at an intensity near the absolute limit of visibility. The observer often mistook the half circle for the full circle, whereas the original expectation had been that sometimes he would merely tend to mistake the barely seen full circle for one of its component halves (Pitres, 1948).
observations with poorer telescopes of lower magnification. They show the uncertainty of interpretations made by scientific observers on the basis of insufficient information, when the true shape of the object is unknown.

In everyday perception there is a tendency to ignore or reject what does not fit in with the expected pattern—as well as to see expected things even if they are not there. G. H. Lewes (1879), quoted by Mrs Johnson Abercrombie (1960), expressed this in saying 'And the new object presented to Senses, or the new idea presented to Thought, must

also be tolerable in old experiences, be re-recognized as like them, otherwise it will be unperceived, uncomprehended.'

In the exceptional cases of great artists taking reality as their theme, however, this vicious circle can be broken. 'Originality', wrote Hazlitt (1778-1830), 'is the seeing nature differently from others, and yet as it is in itself.' Rembrandt, he said, 'lived in and revealed to others a world of his own, and might be said to have invented a new view of nature. He did not discover things out of nature, in fiction or fairytale, or make a voyage to the moon 'to descry new lands, rivers or mountains in her spotty globe', but saw things in nature that everyone had missed before him, and gave others eyes to see them with. This is the test and triumph of originality, not to show us what has never been, and what we may therefore very easily never have dreamt of, but to point out to us what is before our eyes and under our feet, though we had no suspicion of its existence, for want of sufficient strength of intuition, or determined grasp of mind to seize and retain it.' (Hazlitt, 1936.)

This may be contrasted with Pascal's (1623-66) remark: 'What vanity is painting, which attracts admiration by the resemblance of things the originals of which are not admired.' (Pascal, 1659.) While Pascal did not admire the original objects depicted, however, the artist probably admired them. Laymen are often astonished by the kind of things which arouse the wonder of original artists. A great artist expresses his sense of wonder in his works; but there are persons, even most intelligent ones, like Pascal, apparently, who fail to perceive it even in the paintings of great masters.

While my perception of the visual world is limited, yet reality does potentially present us all with an 'infinite variety'. An original artist therefore may become aware of certain visual elements or certain aspects of reality which had not been clearly perceived, or at any rate which had not been represented, by his predecessors. If he succeeds in making use of them in his paintings, he will have made an 'artistic discovery'. What the physicist and astronomer Biot (1774-1862) said of science is applicable to painting: 'Nothing is so easy to see than what has been found yesterday, and nothing more difficult than what will be found tomorrow.' Needless to say, in the art of painting, things have not evolved in a simple manner. Among other things, on account of the limitations of pictorial representation, the utilization of new visual elements may come into a conflict, even from the mere standpoint of realism, with that of elements already in use.

On account of these limitations and of those of our visual perception, a variable period of time is required before the new visual elements which the artist has introduced in his work become understood and accepted by the public. For the artist's contemporaries, such a work may present a character at the same time of strangeness and of truth which captivates their attention. Those who will come later will not be impressed in the same way; on the contrary, what may strike them may be the lack of other elements, more
A painting can fix an aspect of reality

The artist's aim is to communicate something well-defined—even if it is the 'expression of vagueness'. His wish, therefore, must be to facilitate the spectator's perception and to reduce its arbitrariness. It might be questioned whether it is really possible to reach such a result. For, since it is so difficult to see the real world, must not the spectator's perception of the painting itself be submitted to the same degree of uncertainty and variability?

It seems that this is not so, at any rate in the case of a painting seen by the artist's contemporaries at a time in history when there is an accepted style of painting, that is, a general mode of depiction everyone is used to. First, it is clear that the external world is continually changing, as is only too obvious to those who try to paint, say, a landscape on location, whereas the painting does not change in this way. A portrait, for instance, does fix a certain aspect of the sitter. Furthermore, reality is often confusing—and so in fact are many photographs. But the artist, even if he does not actually rearrange his subject, may at leisure choose his standpoint, his lighting and create a clear and coherent picture.1

The main difference between perception of reality and perception of a representational painting, however, is caused by the very limitations of the artist's perception of reality. These limitations must result in a reduction of the play of perception in the spectator who views the artist's work. For even the most realistic paintings can only contain a restricted number of visual elements taken from reality. This is an essential property of all representational art.

Note on the historical evolution of the theory and use of perspective

The method of pictorial representation practised for thousands of years by the Egyptians, for instance, never made use of linear perspective in the full modern conception of the term (Fig. 12.5). Before the middle of the sixteenth century B.C., there are hardly any instances even of the use of foreshortening in the art of any nation. The ancient mode of depiction is, however, based implicitly on precise optical laws, but these laws consist of little more than one single theorem of perspective.

The central projection of a plane figure upon a plane parallel to that of the figure, is a figure which is geometrically similar to the original one (Fig. 12.6). The Egyptians drew separately in this way the main features of the objects, animals and men they wished to represent. These features, each of which can roughly be regarded as flat, were drawn one by one in the plane of the picture, independently of their relative orientation in space—hence the well known fact, for instance that the picture of a human head combines the profile of the head with a frontal view of the eye. This mode of depiction bears some resemblance to the folding of the leaves of a plant specimen when it is pressed to be dried flat before being mounted in a herbarium. The organization of these component parts into a single whole, such as a human body, was to a great extent conventional. There were definite rules to be obeyed in Egyptian paintings and bas-reliefs. These

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1 While the interpretation of the picture of a single object separated from its context often is wildly mistaken when it is presented to a subject under difficult conditions of observation, the interconnections between the various components of a complicated picture by a good artist seem much less liable to misinterpretation under such conditions. Experiments have been made on vision at low luminance levels, in which the observer was shown an engraving by Hogarth ('Hudibras beats Sidrophel and his man Whaum', 1736) in such a dim light that many of the details were impossible to see (Pirenne, Marriott and ODwyer, 1937). Under those conditions, a number of different observers who were merely informed of the general contents of the picture (a magician's cave) all gave of it rather similar descriptions with relatively few mistakes.
12.5 Egyptian limestone bas-reliefs of the XIth Dynasty, c. 2100 B.C.
From the tomb of Tjeti, in Thebes, showing a man and a woman carrying food for the dead man. (British Museum, London.)
Note the life-like realism of the representation of the deities; and the fact that the woman is depicted as having a right hand on her arm.
rules were applied more strictly in the case of important personages than in that of servants, captives and unimportant objects and animals. Again the Egyptian style became freer for a while, during the Amarna period (1375–1355 B.C.), but the ancient Egyptians, and their contemporaries, never used linear perspective to produce a unified picture of a whole scene giving a representation in depth of the component parts with their apparent size and position (Schäfer, 1965; Harris, 1966). Yet it has been seen in Chapter 5 that the main fact of natural perspective, namely the apparent diminution in size of distant objects, was known in the seventh century B.C. and it may be surmised that it was known from prehistoric times.

The Egyptian mode of depiction led to results which may appear very odd to the modern eye, as in the drawing of a woman looking at her reflection in a mirror to paint her lips, reproduced in Fig. 12.7a and b. This method, however, presents the advantage that it gives unambiguous information about the main shape of the constituent parts of the scene or objects represented, rather like certain engineering drawings. Foreshortening, on the other hand, may lead to ambiguities. Thus the paintings on Greek vases of the classical period contain foreshortened, elliptical, representations of warrior’s shields, which might lead to the view that the actual outline of the shields were elliptical, whereas in fact these were circular in shape.

The ancient mode of depiction does not by any means preclude realism; in certain respects many Egyptian and Assyrian paintings and reliefs are most realistic. This is not surprising since the drawing of individual pictorial elements, corresponding to the method

12.6 Central projection of a plane figure on another plane parallel to the plane of the figure. The figure above, which is the projection of $ABCD$ on the plane $EF$ parallel to that of $ABCD$, is geometrically similar to the figure $ABCD$. (From Brook Taylor (1811), New Principles of Linear Perspective.)

12.7a Egyptian drawing of a woman painting her lips.
From the erotic-satirical papyrus, Turin (Cat. No. 2031, XXth Dynasty, c. 1190–1175 B.C.)

12.7b Line drawing made from the original, published by Dr. J. R. Harris in his book Egyptian Art.)
of projection of Fig. 12.6, is in no way conventional. It is interesting to note that, as this method belongs to central perspective, a whole façade may be drawn in exactly the same manner in Renaissance paintings as the individual elements made use of in Egyptian art. This is so for example in many paintings by Piero della Francesca which contain buildings the façade of which is parallel to the picture plane; circles belonging to the façade are then represented as circles, squares as squares.

The profile of the heads appearing on coins is drawn according to the same principle. Held in the right position, the small profile on the coin could be made to coincide visually with the actual profile of the person represented.

This should not be dismissed as an obvious fact, by saying that such circles, squares, human profiles, are simply drawn "as they are". For usually they are much larger in the original than in the representation, which merely is geometrically similar to the original. As Mach (1911) pointed out it is not obvious why two figures which are geometrically similar can look similar to the eye. The empirical reason for this seems to be that they can be superimposed visually on one another; and this ultimately is due to the fact that light travels in straight lines.

To this it might be objected that the same superposition could be achieved for any central projection of the flat object on a surface. But projecting a flat object on a plane parallel to its own plane presents the advantage that when the projection, or the object itself, is placed in front of the eye perpendicularly to the normal direction of sight, its appearance undergoes less alteration when the head moves to a certain extent, than when it is placed in any other position. Thus the outline of a coin held in this way in the frontal plane subtends at the eye a right circular cone which suffers relatively small changes when the position of the eye is slightly changed, whereas the flattened cone which is subtended at the eye by the outline of a coin held inclined to the frontal plane becomes more markedly altered when the position of the centre of rotation of the eye is altered.

The Greeks were the first to introduce perspective into pictorial representation, at any rate in a fragmentary, and possibly purely empirical way. From the end of the sixth century B.C., Greek painted vases show in the drawing of the human body foreshortening and perspective effects of a kind quite unknown in earlier times. Most unfortunately, none of the large-scale works of Greek painters has come down to us. Reflections of these works occur in Greek painted vases of the classical period (Richer, 1966, 1970). But the curved surface of these vases would not lend itself to elaborate perspective effects. On the other hand certain architectural views unearthed at Pompeii (destroyed A.D. 79), are probably based on earlier Greek original paintings, and contain the representation of parallel lines perpendicular to the picture plane, of which many (but not all) converge towards a single point. In two paintings from the Augustan period recently discovered in Rome, such lines converge almost (but not quite) without exception onto one point (Gioseffi, 1966a). The Naples Museum contains an amazing collection of paintings and mosaics of all kinds, from Pompeii and Herculaneum, which strangely foreshadow many of the paintings of the Italian Renaissance. There is no doubt that the main features of modern perspective were fully used, at any rate empirically, in Greco-Roman times.1

There are no texts, however, to prove definitely, to everyone's satisfaction, that the basic principle of perspective, namely the section of the visual pyramid, was fully understood by the artists of the classical Greeks, Hellenistic and Roman periods. Euclid's Optics and his Elements potentially contain all the principles of linear perspective2 but the Optics is essentially a treatise on natural perspective, not on linear perspective, and the section of the visual pyramid (see Lejeune, 1948). On the basis of the whole evidence (texts and paintings still extant) Gioseffi (1966a) concludes that the optical basis of perspective was clearly understood already by the classical Greek painters, and therefore was merely re-discovered at the Renaissance—but this is not universally accepted.

Many of the paintings from Pompeii and Herculaneum contain perspective depictions of round objects such as circular tables, dishes and goblets. Their circular outlines are drawn as more or less regular ellipses. Now there is no indication that the great axes of such ellipses deviate systematically from the horizontal for objects near the edges of the picture, as they should be expected to do according to the strict rules of geometrical perspective (Figs. 9, 13 and 16). Since as a rule Renaissance artists also fail to observe the rules in such cases, however, this does not provide any clear evidence as to whether perspective was used purely empirically (as it seems to have been used by Giorgio and the brothers Van Eyck, for instance) or whether the exact optical theory was known but left unheeded for artistic or other reasons.

The loss of the great works of Greek artists is particularly regrettable, for they would have given most important information on the passage from the pure Egyptian mode of depiction to the elaborate perspective representation which have survived from Greco-Roman times. As has been pointed out, the use of perspective may have tended to come into conflict with the realism already present in the old mode of depiction. Central perspective introduced new elements of realism, of a new, different, kind. It would have been most interesting to know how this change of style, the most momentous change

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1 Dr G. M. A. Richter's (1970) book discusses the development of perspective from about 600 B.C. to A.D. 400 on the basis of the paintings still extant. See also Bradley and Ashtomle (1966).

2 It is not the less strange to consider that the first work giving the solution of the general problem of the vanishing point of a set of parallel straight lines (not parallel to the projection plane) was only published 1900 years after Euclid (Ubald, 1600, Prop. XXXI and Cor. 1). Ubald gives a long geometrical demonstration. A simpler demonstration may be obtained on the basis of the theorem of Euclid's Optics which prove that the angle under which one sees the separation between two parallel lines decreases as the distance from the eye increases.
which ever occurred in the history of painting, exactly took place. It is hardly conceivable that it was quite a sudden one, so that works of the transitional period might have appeared to come as giving the worst of both worlds. It is possible that some of Plato's strictures against the art of his time were connected with this problem (see Schuhl, 1963).

Once perspective had been widely used and accepted, there was no return to the pure pre-Greek mode of representation. Elements of foreshortening remain present in Byzantine and in Western medieval art, even though the paintings as a whole are no longer constructed on the basis of a central projection from one single point (Runain, 1940; Gioseffi, 1966a).

Perspective in the full meaning of the term was discovered (or possibly only rediscovered) in the Renaissance. The Italian architect Brunelleschi (1377–1446) constructed the picture of an architectural view on the principle of the section of the visual pyramid, and arranged it to be viewed in a peep-show so that the spectator's eye was being kept in a fixed position. According to Gioseffi (1966a), Brunelleschi's first experiment was probably made between 1407 and 1409. Here again, unfortunately, our information is secondhand and incomplete: but many other Renaissance texts show that the main optical principle, the section of the visual pyramid, was definitely understood. It appears likely that in Brunelleschi's experiment the picture in perspective, since it was viewed with one eye kept fixed at the centre of projection, was seen in a truly three-dimensional manner, whereas, as has been argued in the present book, the perception or ordinary paintings viewed binocularly in the usual manner is much more complex on account of the spectator's awareness of the painted surface. As shown in previous chapters, this considerable complication was in effect taken into account by Renaissance artists in the depiction of objects having curved surfaces. And it seems clear that it lay at the root, for instance, of Leonardo's struggles with the problem of linear perspective.

In the present century, photography and methods of mechanical pictorial reproduction are playing an immense role in everyone's life. The photographer has to a large extent replaced the artist in the production of pictures serving as records, illustrations, portraits. Again reproductions of an immense variety of works of art of all origins have become readily available in illustrated books. As photography automatically gives pictures in central perspective, there seems to be little doubt that many artists in fact have reacted to this 'unfair competition', preferring to use modes of depiction as different as possible from photographic perspective, and often inspired by works from distant times and places. Yet, perforce, all men continue to see in 'natural perspective'.

1 For further information on the history of perspective, see Carter (1970), Gioseffi (1957, 1957–8, 1965a), Ivins (1933, 1960) and ten Doeschate (1964). The book by Bricke (1978) also contains valuable remarks on perspective, its history, and practical problems such as the hanging and lighting of paintings.